IDENTIFIKACIJA PREMIKOV Z UPORABO RAZLIČNIH GEODETSKIH METOD DEFORMACIJSKE ANALIZE

IDENTIFICATION OF MOVEMENTS USING DIFFERENT GEODETIC METHODS OF DEFORMATION ANALYSIS

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UDK: 528.02
Klasifikacija prispevka po COBISS.SI: 1.02
Prispelo: 20.2.2015
Sprejeto: 17.8.2015

DOI: 10.15292/geodetski-vestnik.2015.03.537-553
REVIEW ARTICLE
Received: 20.2.2015
Accepted: 17.8.2015

SI | EN

IZVLEČEK

V prispevku je predstavljena primerjava metod deformacijske analize geodetskih mrež na podlagi simulacije dvodimenzionalnih komponent vektorjev GNSS v dveh terminskih izmerah za potrebe odkrivanja nedvoumnih premikov v horizontalni ravnini. V članku primerjamo naslednje modele deformacijske analize: Pelzerjevo metodo (hannovrski postopek), metodo Karlsruhe, modificirano metodo Karlsruhe in metodo, ki je izvedena s prostodostopno programsko rešitvijo JAG3D.

ABSTRACT

This paper is based on comparative analysis of applied analysis methods on geodetic networks deformation, based on two-dimensional components of GNSS baseline vectors simulated by two epochs, for the purpose of identifying significant movements in horizontal plane. The following models of deformation analysis have been applied: Pelzer (Hanover procedure) method, Karlsruhe method, Modified Karlsruhe method, and the method implemented in the JAG3D open-source software.

KLJUČNE BESEDE

deformacijska analiza, Pelzerjeva metoda, metoda Karlsruhe, modificirana metoda Karlsruhe, JAG3D

KEY WORDS

Deformation analysis, Pelzer method, Karlsruhe Method, modified Karlsruhe Method, JAG3D

1 INTRODUCTION

Each point on the Earth's surface is subject to constant changes under the influence of various factors, such as tectonic influences, ground waters, landslides earthquakes and other. Constructions developed on the ground are susceptible to subsidence and deformations, occurring as a consequence of internal and external forces, such as the influence of wind, changes in temperature and ground water levels, tectonic and seismic influences, dynamic and static building load, etc. If the deformations had not been accounted for during the design or if they were not discovered timely, they may lead to disturbances in normal utilization, or even to a collapse of the entire construction. In order to prevent negative consequences, it is necessary to observe the construction by applying a geodetic method. Interest in the geodetic methods application for monitoring the construction behavior has suddenly risen after the catastrophic collapse of Gleno Dam in Italy in the year 1923 and of St. Francis Dam in California of 1928 (USA).

The Geodetic Deformation Analysis Methods, referring to the relative construction points' movements, have being applied since 1970's. The most familiar method have been named after development centers or authors: Hanover (Pelzer, 1971), Delft (Heck et al., 1982), Karlsruhe (Heck, 1983), Fredericton (Chrzanowski, 1981) and Munich (Welsch, 1981). Modern application of the methods above can be found with the following authors: (Setan and Singh, 2001; Jäger et al., 2006; Marjetič et al., 2012).

Application of geodetic methods in order to determine land and construction deformation is based on examining of temporal evolution of geodetic networks, realized through physically stabilized points. The proper design of geodetic networks is an integral part of the geodesist's task. Movements of points are being established by comparison of geodetic measurements, being performed in different time epochs. The geodetic network being used to determine movements and deformations consists of the reference points outside the construction (basic network) and of control points on the construction, which are interlinked through geodetic measurements (Caspary, 1987). Basic geodetic network points need to be stabilized outside the expected deformations zone, while the points on the construction need to provide the best possible spatial interpretation of the construction.

2 PELZER METHOD

The method is based on testing congruence of points' coordinates, obtained after network adjustment in the two epochs. Each epoch of measured values is being adjusted independently, with the assumption that the measured values contain only random errors, with a normal distribution.

2.1 Homogenous Accuracy of Measurement in Two Epochs

Adjustment of two epochs provides a posteriori reference variances $\hat{\sigma}_{0_1}^2$ and $\hat{\sigma}_{0_2}^2$. It is therefore necessary to establish whether the measured values in both epochs have homogenous accuracy. For that purpose, we are establishing a zero (H_0) and an alternative (H_a) hypothesis (Pelzer, 1971; Mihailović and Aleksić, 1994, 2008; Ambrožič, 2001; Ašanin, 2003; Vrce, 2011):

$$H_0 : E\left(\hat{\sigma}_{0_1}^2\right) = E\left(\hat{\sigma}_{0_2}^2\right) = \sigma_0^2 \text{ against } H_a : E\left(\hat{\sigma}_{0_1}^2\right) \neq E\left(\hat{\sigma}_{0_2}^2\right) \neq \sigma_0^2 . \quad (1)$$

Tests statistics is:

$$T=\frac{\hat{\sigma}_{0_1}^2}{\hat{\sigma}_{0_2}^2}\sim F_{f_1,f_2}\ \text{for}\ \hat{\sigma}_{0_1}^2>\hat{\sigma}_{0_2}^2\ \text{or}\ T=\frac{\hat{\sigma}_{0_2}^2}{\hat{\sigma}_{0_1}^2}\sim F_{f_1,f_2}\ \text{for}\ \hat{\sigma}_{0_2}^2>\hat{\sigma}_{0_1}^2 \tag{2}$$

where f_1 and f_2 are number of freedom degrees in zero and control measurement epoch.

If the $T\leq F_{f_1,f_2,1-\alpha/2}$, zero hypothesis is not rejected and unique a posteriori variance is being calculated:

$$\hat{\sigma}_0^2=\frac{f_1\cdot\hat{\sigma}_{0_1}^2+f_2\cdot\hat{\sigma}_{0_2}^2}{f} \tag{3}$$

where the f is a sum of numbers of freedom degrees in zero and control measurement epoch.

2.2 Testing the Network Congruence in Two Epochs

Network congruence is tested using mathematical statistics testing methods. For that purpose, the zero (H_0) and the alternative (H_a) hypothesis is being set (Pelzer, 1971; Mihailović and Aleksić, 1994, 2008; Ambrožič, 2001; Ašanin, 2003; Vrce, 2011):

$$H_0:E(\hat{\mathbf{x}}_1)=E(\hat{\mathbf{x}}_2)\ \text{against}\ H_a:E(\hat{\mathbf{x}}_1)\neq E(\hat{\mathbf{x}}_2) \tag{4}$$

where $\hat{\mathbf{x}}_1$ and $\hat{\mathbf{x}}_2$ are vectors of zero and control measurement epoch coordinates.

Average non-fitting θ^2, which contains information on movements of points, is calculated using the formula:

$$\theta^2=\frac{\mathbf{d}^T\cdot\mathbf{Q}_\mathbf{d}^+\cdot\mathbf{d}}{h},\mathbf{d}=\hat{\mathbf{x}}_2-\hat{\mathbf{x}}_1, \tag{5}$$

where: $\mathbf{d}$ – vector of coordinate differences, $\mathbf{Q}_\mathbf{d}^+$ – pseudo-inversion of coordinate differences cofactor matrix, and h – rank of the coordinate differences cofactor matrix $\mathbf{Q}_\mathbf{d}$.

The test statistics is as following:

$$T=\frac{\theta^2}{\hat{\sigma}_0^2}\sim F_{h,f}\,. \tag{6}$$

When the $T\leq F_{h,f,1-\alpha}$, zero hypothesis is not being rejected, i.e. so that the network is congruent in two epochs; then an alternative hypothesis will be accepted.

2.3 Testing Congruence of Basic Network Points

If unstable points' existence in the network is established, a hypothesis on basic network points' congruence is being tested. Hypotheses are being set as (Pelzer, 1971; Mihailović and Aleksić, 1994, 2008; Ambrožič, 2001; Ašanin, 2003; Vrce, 2011):

$$H_0:E(\hat{\mathbf{x}}_{S1})=E(\hat{\mathbf{x}}_{S2})\ \text{against}\ H_a:E(\hat{\mathbf{x}}_{S1})\neq E(\hat{\mathbf{x}}_{S2}) \tag{7}$$

Where $\hat{\mathbf{x}}_{S1}$ and $\hat{\mathbf{x}}_{S2}$ are the vectors of basic points' coordinates in the zero and control measurement epoch.

Vector of the coordinate differences **d** and the pseudo-inversion of coordinate differences cofactor matrix $\mathbf{Q}_{\mathbf{d}}^{+}$ are decomposed as follows:

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_S \\ \mathbf{d}_O \end{bmatrix} \qquad \mathbf{Q}_{\mathbf{d}}^{+} = \mathbf{P}_{\mathbf{d}} = \begin{bmatrix} \mathbf{P}_{SS} & \mathbf{P}_{SO} \\ \mathbf{P}_{OS} & \mathbf{P}_{OO} \end{bmatrix} \tag{8}$$

where the *S* designates the basic network points and *O* designates the points representing the construction.

Average lack of coordination for basic network points is calculated using following formula:

$$\theta_S^2 = \frac{\mathbf{d}_S^T \cdot \overline{\mathbf{P}}_{SS} \cdot \mathbf{d}_S}{h_S} \tag{9}$$

where $\overline{\mathbf{P}}_{SS} = \mathbf{P}_{SS} - \mathbf{P}_{SO} \cdot \mathbf{P}_{OO}^{-1} \cdot \mathbf{P}_{OS}$.

The test statistics reads:

$$T = \frac{\theta_S^2}{\hat{\sigma}_0^2} \sim F_{h_S, f} \tag{10}$$

where the h_S is a rank of $\overline{\mathbf{P}}_{SS}$.

When the $T \leq F_{h_S, f, 1-\alpha}$, zero hypothesis is not being rejected, i.e. basic network points are congruent in two epochs or else; when the (6) alternative hypothesis is accepted.

2.4 Localization of Unstable Basic Points

When the basic network points' congruence in two epochs has not been established, there is a need to localize unstable basic points, i.e. to determine which basic points are unstable. For that purpose, coordinate differences vector $\mathbf{d}_S$ and coordinate differences cofactor matrix $\overline{\mathbf{P}}_{SS}$ are decomposed as follows (Pelzer, 1971; Mihailović and Aleksić, 1994, 2008; Ambrožič, 2001; Ašanin, 2003; Vrce, 2011):

$$\mathbf{d}_S = \begin{bmatrix} \mathbf{d}_F \\ \mathbf{d}_B \end{bmatrix} \qquad \overline{\mathbf{P}}_{SS} = \begin{bmatrix} \mathbf{P}_{FF} & \mathbf{P}_{FB} \\ \mathbf{P}_{BF} & \mathbf{P}_{BB} \end{bmatrix} \tag{11}$$

where the *F* designates basic points being considered conditionally stable and the *B* designates basic points considered conditionally unstable.

For each basic network point, average gap is set as following:

$$\theta_j^2 = \frac{\overline{\mathbf{d}}_{B_j}^T \cdot \mathbf{P}_{BB_j} \cdot \overline{\mathbf{d}}_{B_j}}{h_{B_j}}, (j = 1, 2, \ldots, k) \tag{12}$$

where $\overline{\mathbf{d}}_B = \mathbf{d}_B + \mathbf{P}_{BB}^{-1} \cdot \mathbf{P}_{BF} \cdot \mathbf{d}_F$.

In the set of values , the maximum value is recognized, so that the point to which the maximum value refers to is considered unstable and rejected (left out) from the set of basic points, which are still considered to be conditionally stable points.

Average mismatch for the remaining $k-1$ basic points is being determined as:

$$\theta_{REST}^{2}=\frac{\mathbf{d}_{F}^{T}\cdot\overline{\mathbf{P}}_{FF}\cdot\mathbf{d}_{F}}{h_{F}} \tag{13}$$

where $\overline{\mathbf{P}}_{FF}=\mathbf{P}_{FF}-\mathbf{P}_{FB}\cdot\mathbf{P}_{BB}^{-1}\cdot\mathbf{P}_{BF}$ and h_F is rank of $\overline{\mathbf{P}}_{FF}$.

The test statistics reads:

$$T=\frac{\theta^{2}{}_{REST}}{\hat{\sigma}_{0}^{2}}\sim F_{h_F,f}\,. \tag{14}$$

If the $T\leq F_{h_F,f,1-\alpha}$, zero hypothesis is not rejected, i.e. it is considered that all $k-1$ basic points are stable. If the $T>F_{h_F,f,1-\alpha}$, an alternative hypothesis is being accepted, i.e. there are still some unstable points among $k-1$ basic points, when it is necessary to identify unstable points in the manner shown.

2.5 Testing Congruency of Construction Points

To test movements of construction points, the coordinate differences vector **d** and pseudo-inversion of coordinate differences cofactor matrix $\mathbf{Q}_{\mathbf{d}}^{+}$ are decomposed as follows (Pelzer, 1971; Mihailović and Aleksić, 1994, 2008; Ambrožič, 2001; Ašanin, 2003; Vrce, 2011):

$$\mathbf{d}=\begin{bmatrix}\mathbf{d}_{F}\\ \mathbf{d}_{O}\end{bmatrix}\qquad \mathbf{Q}_{\mathbf{d}}^{+}=\mathbf{P}_{\mathbf{d}}=\begin{bmatrix}\mathbf{P}_{FF} & \mathbf{P}_{FO}\\ \mathbf{P}_{OF} & \mathbf{P}_{OO}\end{bmatrix} \tag{15}$$

where the F designates basic points identified as stable, and O designates the basic points identified as unstable and construction points.

Average mismatch is determined using the formula:

$$\theta_{O}^{2}=\frac{\overline{\mathbf{d}}_{O}^{T}\cdot\mathbf{P}_{OO}\cdot\overline{\mathbf{d}}_{O}}{h_{O}} \tag{16}$$

where $\overline{\mathbf{d}}_{O}=\mathbf{d}_{O}+\mathbf{P}_{OO}^{-1}\cdot\mathbf{P}_{OF}\cdot\mathbf{d}_{F}$ and h_O is rank of $\mathbf{P}_{OO}$.

The test statistics reads:

$$T=\frac{\theta_{0}^{2}}{\hat{\sigma}_{0}^{2}}\sim F_{h_O,f}\,. \tag{17}$$

If the $T\leq F_{h_O,f,1-\alpha}$, zero hypothesis is not rejected; or else an alternative hypothesis is being accepted.

3 KARLSRUHE METHOD

Karlsruhe method is based on independent adjustment of zero and the control epoch and also on their joint adjustment. In the first phase, measured values in the individual epochs are adjusted using the Least Squares Method (LSM). In the second phase, measured values in zero and control epoch are jointly adjusted. Joint adjustment of the two epochs is being done under the assumption that the basic points are congruent in two epochs and that the network scale is the same in both epochs.

In the first phase, adjustment is made independently for all measured values in each epoch, using indirect adjustment method:

$$\mathbf{v}_i = \mathbf{A}\cdot\mathbf{x}_i + \mathbf{f}_i \;, i = 1,2,\ldots,k \tag{18}$$

where k is the number of epochs. The network may be adjusted usually or with the minimum trace.

From each individual adjustment, a square form Ω_i is being determined, and joint square form for all epochs is obtained by summing square forms of individual epochs' adjustments:

$$\Omega_0 = \sum_{i=1}^{k}\Omega_i = \sum_{i=1}^{k}\mathbf{v}_i^T\cdot\mathbf{P}_i\cdot\mathbf{v}_i = \mathbf{v}^T\cdot\mathbf{P}\cdot\mathbf{v}\;. \tag{19}$$

Total number of the degrees of freedom b is obtained by summing up the degrees of freedom b_i ($b_i = n_i - u_i$) from individual epochs' adjustments.

During the second phase, measured values in zero and control measurement epoch are jointly adjusted. In joint adjustment of the two epochs, vector of unknown coordinates is divided into three sub vectors:

$$\mathbf{x}^T = (\mathbf{z}^T, \mathbf{x}_1^T, \mathbf{x}_2^T) \tag{20}$$

where:

$\mathbf{z}$ – sub vector of basic points is assumed to be stable in both epochs;

$\mathbf{x}_1$, $\mathbf{x}_2$ – sub vectors of construction points or points assumed to be unstable.

From the joint adjustment, square form Ω_z is being determined, containing information on measurement errors and movements of unstable points. Joint adjustment square form Ω_z gets deducted from square form Ω_0, which contains information about measurement errors only (Heck, 1983; Ninkov, 1985; Mihailović and Aleksić, 1994, 2008; Ambrožič, 2004):

$$\Omega_h = \Omega_z - \Omega_0 = \mathbf{v}_z^T\cdot\mathbf{P}\cdot\mathbf{v}_z - \mathbf{v}^T\cdot\mathbf{P}\cdot\mathbf{v}\;. \tag{21}$$

The new square form Ω_h contains only the information about unstable points' movement.

Test statistics (Heck, 1983; Ninkov, 1985; Mihailović and Aleksić, 1994, 2008; Ambrožič, 2004):

$$F = \frac{\Omega_h / f}{\Omega_0 / b} = \frac{\left(\mathbf{v}_z^T\cdot\mathbf{P}\cdot\mathbf{v}_z - \mathbf{v}^T\cdot\mathbf{P}\cdot\mathbf{v}\right)}{\mathbf{v}^T\cdot\mathbf{P}\cdot\mathbf{v}}\cdot\frac{b}{f} \tag{22}$$

where: $f = (k-1)\cdot n\cdot p_0 - d$, k – is number of epochs, n – is the geodetic network dimension, p_0 – is the number of conditionally stable points, d – rank defect of matrix $\mathbf{A}$.

If the $F \le F_{f,b,1-\alpha}$, zero hypothesis is not rejected, i.e. all of the points from conditionally stable points are indeed stable points; or else an alternative hypothesis is being accepted.

3.1 Determining Unstable Points in the Set of Conditionally Stable Points

With $F > F_{f,b,1-\alpha}$, the set of conditionally stable points contains unstable points. It is necessary to determine such points. For that purpose, joint adjustments are repeated from which one conditionally stable point is being excluded successively. The adjustment providing the minimum value of the square form

$\Omega_{z,min}$ indicates that the point excluded from the adjustment is to be considered unstable. That point is definitely excluded from the set of conditionally stable points, and the entire procedure is repeated without it. The procedure is repeated iteratively, until the condition $F \leq F_{f,b,1-\alpha}$, is met, and the points left in the set of conditionally stable points afterwards are considered stable.

3.2 Localization of Deformations

Deformations are being localized for each point. The zero hypothesis assumes that the point T_j did not move, while the alternative hypothesis assumes that the point T_j did move. Zero (H_0) and alternative (H_a) hypotheses are being set (Heck, 1983; Ninkov, 1985; Mihailović and Aleksić, 1994, 2008; Ambrožič, 2004):

$$H_0 : E\left(\hat{\mathbf{d}}_j\right) = 0 \qquad \text{against} \qquad H_a : E\left(\hat{\mathbf{d}}_j\right) \neq 0 . \tag{23}$$

The test statistics reads:

$$F_j = \frac{\theta_j^2}{\hat{\sigma}_0^2} = \frac{\hat{\mathbf{d}}_j^T \cdot \mathbf{Q}_{\hat{\mathbf{d}}_j}^{-1} \cdot \hat{\mathbf{d}}_j}{m \cdot \hat{\sigma}_0^2} \sim F_{m,f} \tag{24}$$

where:

$$\mathbf{Q}_{\hat{\mathbf{d}}_j} = \mathbf{B}^T \cdot \mathbf{Q}_{\hat{\mathbf{x}}} \cdot \mathbf{B} = \begin{bmatrix} q_{\hat{y}\hat{y}_{\hat{d}}} & q_{\hat{y}\hat{x}_{\hat{d}}} \\ q_{\hat{x}\hat{y}_{\hat{d}}} & q_{\hat{x}\hat{x}_{\hat{d}}} \end{bmatrix},$$

$$\hat{\mathbf{d}}_j = \mathbf{B}^T \cdot \hat{\mathbf{x}} = \begin{bmatrix} \hat{y}_{2,j} - \hat{y}_{1,j} \\ \hat{x}_{2,j} - \hat{x}_{1,j} \end{bmatrix},$$

$$\hat{\sigma}_0^2 = \frac{b_1 \cdot \hat{\sigma}_{0_1}^2 + b_2 \cdot \hat{\sigma}_{0_2}^2}{f}, f = b_1 + b_2 ,$$

m – geodetic network dimension.

If the $F_j \leq F_{m,f,1-\alpha}$, zero hypothesis is not rejected, i.e. the point is stable; or else an alternative hypothesis is being accepted.

4 MODIFIED KARLSRUHE METHOD

The method consists of free adjustment zero measurement epoch using the LSM with minimization of the part of the trace matching the assumed stable points. Adjustment of control measurement epoch is also being done using the LSM with minimization of the part of trace matching the assumed stable points; by accepting coordinates of adjusted zero measurement epoch as the approximate coordinates of the control measurement epoch (Ninkov, 1985).

Points' stability control is being done using the Unimodal Transformation Method (without changing the scale). After estimating transformation parameters under the LSM, the control of differences of the transformed control epoch coordinates and of zero epoch coordinates difference is under threshold of double coordinate standards from the control epoch adjustment. In the event of discovered unstable points, it is necessary to perform new adjustments by minimizing the part of a trace matching confirmed stable points.

After adjustment, deformations are being localized for each point individually. In order to test the stability of points, zero (H_0) and alternative (H_a) hypothesis are being set (Ninkov, 1985):

$$H_0 : E\left(\hat{\mathbf{d}}_j\right) = 0 \qquad \text{against} \qquad H_a : E\left(\hat{\mathbf{d}}_j\right) \neq 0 \cdot \tag{25}$$

The test statistics reads:

$$F_j = \frac{\theta_j^2}{\hat{\sigma}_0^2} = \frac{\hat{\mathbf{d}}_j^T \cdot \mathbf{Q}_{\hat{\mathbf{d}}_j}^{-1} \cdot \hat{\mathbf{d}}_j}{2 \cdot \hat{\sigma}_0^2} \sim F_{2,f} \tag{26}$$

If the $F_j \leq F_{2,f,1-\alpha}$, zero hypothesis is not rejected; or else an alternative hypothesis is being accepted.

The presented statistical testing of hypothesis may also be interpreted geometrically. In the event of zero hypothesis (H_0), formula (26) may be written as follows:

$$\hat{\mathbf{d}}_j^T \cdot \mathbf{Q}_{\hat{\mathbf{d}}_j}^{-1} \cdot \hat{\mathbf{d}}_j \leq 2 \cdot \hat{\sigma}_0^2 \cdot F_{2,f,1-\alpha} \cdot \tag{27}$$

If we replace ≤ with = in the formula (27), the expression becomes an ellipse equation, which is identical to the relative error ellipse between the points $T_{1,j}$ and $T_{2,j}$ increased for the factor $\sqrt{2 \cdot F_{2,f,1-\alpha}}$. Axis of the relative error ellipses are calculated using the formula:

$$A_i = \sqrt{\hat{\sigma}_0^2 \cdot 2 \cdot F_{2,f,1-\alpha} \cdot \lambda_i}, (i = 1,2) \cdot \tag{28}$$

The deformation analysis using relative error ellipses can be presented graphically with ease (Figure 1). Point movement vectors and increased relative error ellipses are drawn on the same sketch. The movement vector $\hat{\mathbf{d}}_j$ starts at the point $T_{1,j}$ towards the point $T_{2,j}$, and the point $T_{2,j}$ is the center of ellipse. If the point $T_{1,j}$ is outside the ellipse surface, zero hypothesis is being rejected, i.e. deformations in point $T_{1,j}$ are accepted with a certain probability.

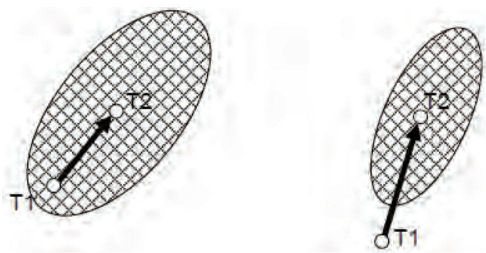

Figure 1: Deformations analysis using relative errors ellipses.

Increments of approximate (adjusted) coordinates in the adjustment process of spatial movement components are calculated as follows (Ninkov, 1985):

$$d_{y_j} = \hat{Y}_{2_j} - \hat{Y}_{1_j}, \quad d_{x_j} = \hat{X}_{2_j} - \hat{X}_{1_j} \, . \tag{29}$$

Standards for determining spatial movements' components are approximately equal to the square root of the sum of coordinates' determination squares.

5 JAG3D METHOD

JAG3D is Open-Source Software developed for the geodetic networks adjustment and deformation analysis, by Michael Lösler (URL1). In the first phase, both the zero and control measurement epoch are independently adjusted using indirect adjustment method. Testing measurement accuracy homogeneity is being done in the manner explained in Pelzer Method. In the next step, congruency of the basic network points is being tested. If basic points' set contains unstable points, such unstable basic points are being localized. Zero (H_0) and alternative (H_a) hypotheses are being set:

$$H_0 : E\left(\nabla_{R,j}\right) = 0 \qquad \text{against} \qquad H_a : E\left(\nabla_{R,j}\right) \neq 0 . \tag{30}$$

The test statistics is formed (URL2):

$$T_{prio,j} = \frac{\nabla^T_{R,j} \cdot \mathbf{Q}^{-1}_{\nabla\nabla R,j} \cdot \nabla_{R,j}}{m \cdot \sigma_0^2} \sim F_{m,\infty} \quad \text{– a priori test statistics} \tag{31}$$

or

$$T_{post,j} = \frac{\nabla^T_{R,j} \cdot \mathbf{Q}^{-1}_{\nabla\nabla R,j} \cdot \nabla_{R,j}}{m \cdot \hat{\sigma}_j^2} \sim F_{m,f-m} \quad \text{– a posteriori test statistics} \tag{32}$$

where:

$\nabla_{R,j}$ – is movement vector;
$\mathbf{Q}_{\nabla\nabla R,j}$ – is cofactor matrix of movement estimate;
m – is geodetic network dimension;
σ_0^2 – is a priori variance;
$\hat{\sigma}_j^{\,2} = \dfrac{\Omega - \nabla^T_{R,j} \cdot \mathbf{Q}^{-1}_{\nabla\nabla R,j} \cdot \nabla_{R,j}}{f-m}$ – is reduced a posteriori variance.

If the $T_{prio,j} < F_{m,\infty}$ or $T_{post,j} < F_{m,f-m}$, zero hypothesis is not rejected, i.e. the point is stable; or else an alternative hypothesis is being accepted. Gauss-Markov model of the joint adjustment of both epochs is (URL2):

$$\begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{R,1} & \mathbf{A}_{O,1} & 0 \\ \mathbf{A}_{R,2} & 0 & \mathbf{A}_{O,2} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x}_R \\ \mathbf{x}_{O,1} \\ \mathbf{x}_{O,2} \end{bmatrix} \tag{33}$$

where:

$\mathbf{l}_1$ – is the vector of zero measurement epoch measured values;
$\mathbf{l}_2$ – is the vector of control measurement epoch measured values;
$\mathbf{x}_R$ – is the sub vector of basic points with stability confirmed (conditionally stable points);
$\mathbf{x}_{O,1}$, $\mathbf{x}_{O,2}$ – is the sub vector of points on construction or points considered conditionally unstable.

After joint adjustment, unstable points on construction are localized. In order to localize unstable points on construction, zero (H_0) and alternative (H_a) hypothesis are being set:

$$H_0 : E\left(\mathbf{d}_k\right) = 0 \qquad \text{against} \qquad H_a : E\left(\mathbf{d}_k\right) \neq 0 . \tag{34}$$

The test statistics reads (URL2):

$$T_{prio,k} = \frac{\mathbf{d}_k^T \cdot \mathbf{Q}_{\mathbf{dd},k}^{-1} \cdot \mathbf{d}_k}{m \cdot \sigma_0^2} \sim F_{m,\infty} \quad \text{– a priori test statistics} \tag{35}$$

or

$$T_{post,k} = \frac{\mathbf{d}_k^T \cdot \mathbf{Q}_{\mathbf{dd},k}^{-1} \cdot \mathbf{d}_k}{m \cdot \hat{\sigma}_0^2} \sim F_{m,f-m} \text{ – a posteriori test statistics} \tag{36}$$

where:

$$\mathbf{d}_k = \mathbf{F} \cdot \begin{bmatrix} \mathbf{x}_{O,1} \\ \mathbf{x}_{O,2} \end{bmatrix} \quad \text{– vector of movement;}$$

$$\mathbf{Q}_{\mathbf{dd},k} = \mathbf{F} \cdot \begin{bmatrix} \mathbf{Q}_{\mathbf{x}_{O,1},\mathbf{x}_{O,1}} & \mathbf{Q}_{\mathbf{x}_{O,1},\mathbf{x}_{O,2}} \\ \mathbf{Q}_{\mathbf{x}_{O,2},\mathbf{x}_{O,1}} & \mathbf{Q}_{\mathbf{x}_{O,2},\mathbf{x}_{O,2}} \end{bmatrix} \cdot \mathbf{F}^T \quad \text{– movement estimate cofactor matrix;}$$

$$\mathbf{F} = \begin{bmatrix} 0 & \dots & -\mathbf{I}_k & \dots & \mathbf{I}_k & \dots & 0 \end{bmatrix}.$$

If the $T_{prio,\,k} < F_{m,\,\infty}$ or $T_{post,\,k} < F_{m,\,f-m}$, zero hypothesis is not rejected, i.e. the point is stable; otherwise an alternative hypothesis is being accepted.

6 CALCULATION EXAMPLE

For the purposes of applying Deformation Analysis Methods described in the paper, the geodetic control GNSS 2D network was simulated, consisting of 9 points. Basic network points are points 1, 2, 3 and 4, while points 5, 6, 7, 8 and 9 interpret the construction (Figure 2). All observations and deformations in the network are simulated. The simulated vectors are linearly independent, and the standard deviation of two-dimensional component of GNSS baseline vector is 5 mm + 0.5 ppm. Approximate points' coordinates and simulated deformations are shown in Table 1, with the data on two epoch vector observations being given in Table 2, while the network sketch is shown in Figure 2.

Table 1: Approximate points' coordinates and simulated deformations.

Point number	Y [m]	X [m]	d_i [mm]	ν_i [°]
1	1320	1400	--	--
2	1370	1270	--	--
3	1650	1125	--	--
4	1670	1310	--	--
5	1785	1250	--	--
6	1740	1400	15	218
7	1625	1530	40	225
8	1470	1585	10	197
9	1325	1570	5	182

Table 2: Two epoch vector observation data.

Vectors		Zero epoch		Control epoch	
From	To	ΔY [m]	ΔX [m]	ΔY [m]	ΔX [m]
1	2	50.00290	-129.9970	49.9975	-129.9990
1	3	330.0092	-275.0010	330.0024	-275.0010
1	5	465.0007	-149.9990	465.0031	-150.0020
1	4	349.9914	-89.9941	350.0097	-89.9971
1	6	420.0068	-0.0091	419.9920	-0.0065
1	7	305.0051	129.9935	304.9722	129.9799
1	8	149.9977	185.0017	149.9947	184.9907
1	9	5.0009	169.9974	4.9991	169.9957
2	3	280.0031	-145.0010	279.9998	-145.0025
2	5	415.0015	-20.0060	414.9930	-19.9936
2	4	300.0028	39.9964	299.9990	39.9997
2	6	369.9995	129.9984	369.9938	129.9863
2	7	254.9998	259.9971	254.9697	259.9802
2	8	100.0004	314.9965	99.9990	314.9937
2	9	-45.0032	299.9983	-44.9962	299.9979
2	1	-49.9943	129.9962	-49.9957	129.9960
3	5	135.0000	125.0054	135.0031	125.0017
3	6	90.0027	274.996	89.9818	274.9902
3	4	19.9966	185.0073	19.9977	184.9956
3	7	-25.0043	404.9985	-25.0244	404.9705
3	8	-180.0000	459.9942	-180.0014	459.9888
3	9	-325.001	444.9975	-325.0001	444.9971
3	1	-329.995	275.0037	-329.9983	274.9960
3	2	-279.998	145.0042	-279.9955	145.0044
4	3	-19.9978	-184.999	-19.9994	-184.9929
4	5	114.9942	-59.9989	114.9970	-60.0012
4	6	69.9961	90.0025	69.9901	89.9881
4	7	-44.9992	219.9942	-45.0288	219.9735
4	8	-200.0010	274.9977	-199.9994	274.9953
4	9	-344.9950	259.9927	-344.9985	259.9971
4	1	-349.9950	89.9982	-349.9978	90.0016
4	2	-300.0020	-39.9962	-299.9994	-40.0019

Zero and control measurement epoch were adjusted, along with joint adjustment of both measurement epochs. Datum was defined by minimum trace to the basic network points. Adjustments were performed using the JAG3D and MatGeo (developed at the Faculty of Technical Sciences in Novi Sad, using Matlab environment) software. Deformation analysis by means of the JAG3D Method was performed using the JAG3D software, while deformation analysis using other methods was implemented in the MatGeo program. The significance level used in calculations is $\alpha = 0.05$.

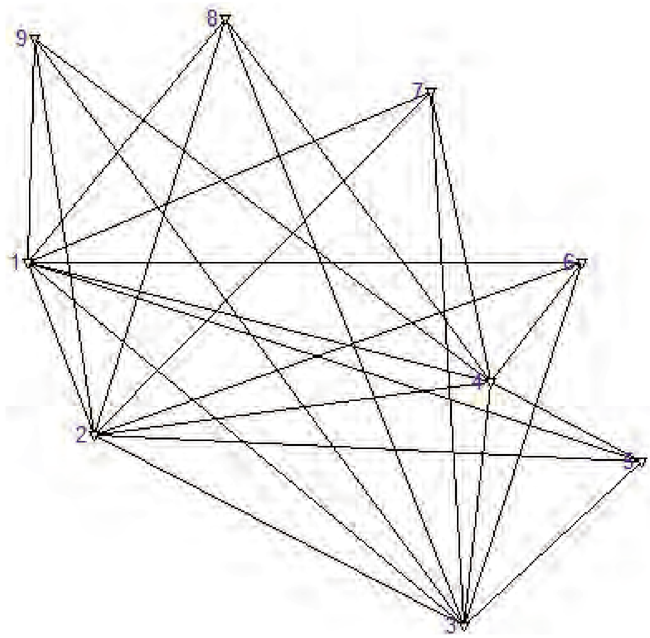

Figure 2: Network sketch in the scale of 1:5000.

The value $\sigma_0 = 1$ was accepted as a priori standard deviation. Following values were obtained from the adjustment of zero and control measurement epochs: $\hat{\sigma}_{0_1} = 1.095$, $\hat{\sigma}_{0_2} = 1.023$, $\Omega_0 = 57.527$, $\Omega_1 = 50.212$, $f_0 = 48$ and $f_1 = 48$. Following values were obtained from adjustment of both epochs: $\hat{\sigma}_{0_z} = 1.059$, $\Omega_z = 114.387$ and $f_z = 102$. Adjusted coordinates of zero and control epoch measurements are shown in Table 3.

Table 3: Adjusted points' coordinates from zero and control measurement epoch.

Point number	Zero epoch		Control epoch	
	Y [m]	X [m]	Y [m]	X [m]
1	1320.0001	1399.9995	1319.9999	1399.9992
2	1369.9995	1270.0017	1370.0000	1270.0001
3	1650.0011	1124.9983	1650.0000	1125.0011
4	1669.9993	1310.0004	1670.0002	1309.9996
5	1784.9991	1250.0004	1784.9991	1250.0013
6	1740.0012	1399.9970	1739.9893	1399.9895
7	1625.0003	1529.9958	1624.9722	1529.9761
8	1469.9991	1584.9976	1469.9982	1584.9921
9	1325.0004	1569.9965	1325.0011	1569.9969

6.1 Pelzer Method

In the first step, homogeneity of measurement accuracy in two epochs is being tested. For that purpose, the test statistics is being formed according to the formula (2), while the test statistics $T = 1.146$ is lower than critical value $F_{48,48,0.975} = 1.773$, so that the values measured in two epochs have homogeneous accuracy. Unified degrees of freedom from two epochs are $f = 96$. Unified standard deviation from two epochs is calculated using the formula (3) being $\hat{\sigma}_0 = 1.059$.

In the second step, the network congruency in two epochs is being tested. According to the formula (6), test statistics is thus calculated: $T = 12.400$, so that the value obtained is greater than the critical value $F_{16,96,0.95} = 1.750$, with the conclusion that network points' coordinates from two epochs are not congruent.

In this step, congruency of the basic network points is being tested. The test statistics $T = 0.987$, being calculated using formula (10), is lower than critical value $F_{6,96,0.95} = 2.195$, thus the conclusion can be made that the basic network points from two epochs are congruent.

After congruency of basic network points had been established, stability of construction points is being tested. Test statistics $T = 19.248$, determined using formula (17), is greater than critical value $F_{10,96,0.95} = 1.931$, thus the conclusion can be made that the construction points' coordinates from two epochs are not congruent.

When non-congruency of construction points is established, unstable construction points are being localized next. For each point, average mismatch θ_i^2 is being calculated using formula (12) (Table 4).

Localization procedure is done through iterations. In the first iteration, point 7 is identified as unstable (Table 4), and is being excluded from the set of construction points.

Table 4: Localization of unstable construction points by Pelzer Method.

Point number	Iteration 1	Iteration 2	Movement	Slope
	θ_i^2	θ_i^2	d_i [mm]	ν_i [°]
5	0.066	0.066	0.919	0.623
6	15.088	**15.088**	14.029	238.224
7	**90.543**	--	34.313	235.004
8	2.264	2.264	5.487	189.325
9	0.048	0.048	0.794	54.713

In order to test the stability of the remaining construction points, the test statistics is being calculated using the formula (14). In the set of remaining points, some are unstable since the test statistics $T = 3.891$ is greater than critical value $F_{8,96,0.95} = 2.036$. In the second iteration, the point number 6 has the greatest value of average mismatch values and is thus considered as unstable. It is necessary to test the stability of the remaining construction points. The set of remaining points does not contain unstable points, since the test statistics reads $T = 0.706$, being lower than critical value $F_{6,96,0.95} = 2.195$. Discovered movements of points are shown in Table 4. Points 6 and 7 are identified as unstable, while the remaining points are identified as stable.

6.2 Karlsruhe Method

The set of conditionally stable points are the basic network of points. In order to test the stability of conditionally stable points, the test statistics is calculated using the formula (22). Test statistics of the $F = 0.987$ is lower than the critical value $F_{6,96,0.95} = 2.195$, thus the conclusion can be made that the set of conditionally stable points contains no unstable points. Localization of conditionally unstable points is being performed by calculating test statistics for each point using the formula (24). Table 5 shows values of test statistics *F* and critical values *F* for each point. If test statistics *F* is lower than the critical value *F*, the point is stable; otherwise it is unstable. Using Karlsruhe Method, points 6 and 7 are identified as unstable. Points' movements are shown in Table 5.

Table 5: Localization of unstable points by Karlsruhe Method.

Point number	F	$F_{2,96,0.95}$	d_i [mm]	v_i [°]
5	0.059	3.091	0.919	0.623
6	**13.454**	3.091	14.029	238.224
7	**80.738**	3.091	34.313	235.004
8	2.018	3.091	5.487	189.325
9	0.043	3.091	0.794	54.713

6.3 Modified Karlsruhe Method

Basic network points' congruency has been verified using the Unimodal transformation. In the next step, localization of unstable points is being performed. For each point, value of *F* test statistics is calculated using formula (26), and the values obtained are shown in Table 6. If tested *F* statistics is lower than *F* critical value, the point is stable; otherwise the point is unstable. Using this method, points 6 and 7 were identified as unstable. Points' movements discovered are shown in Table 6.

Table 6: Localization of unstable points by modified Karlsruhe Method.

Point number	F	$F_{2,48,0.95}$	d_i [mm]	v_i [°]
1	0.055	3.191	0.431	218.454
2	0.849	3.191	1.684	163.821
3	2.625	3.191	2.987	339.197
4	0.427	3.191	1.197	133.983
5	0.058	3.191	0.909	0.630
6	**13.469**	3.191	14.042	238.155
7	**80.746**	3.191	34.329	234.977
8	2.033	3.191	5.510	189.291
9	0.042	3.191	0.782	56.514

6.4 JAG3D Method

After independent adjustment of zero and of control measurement epochs, the testing of basic network points' congruency in two epochs has been performed. Basic network points' congruency in two epochs has been determined. In the second step, the joint adjustment of zero and of the control measurement epoch has been performed, with basic network points being included in the set of conditionally stable points. After joint adjustment of both epochs, localization of unstable construction points has been performed. For each point, test statistics for the T has been calculated using formula (35) and (36).

Table 7: Localization of unstable points, JAG3D Method.

Point number	T_{prio}	T_{post}	$T_{2,\infty,0.95}$	$T_{2,100,0.95}$	d_i [mm]	v_i [°]
5	0.066	0.059	2.996	3.087	0.919	0.623
6	**15.088**	**13.454**	2.996	3.087	14.029	238.202
7	**90.543**	**80.738**	2.996	3.087	34.313	235.004
8	2.264	2.018	2.996	3.087	5.487	189.342
9	0.048	0.043	2.996	3.087	0.794	54.713

Values of test statistics for each point are shown in Table 7. If the test statistics for the T is lower than the F critical value, the point is stable; otherwise the point is unstable. Using the method implemented in the JAG3D software, points 6 and 7 were identified as unstable. The discoverent points' movements are shown in Table 7.

7 ANALYSIS OF COMPARATIVE RESULTS

The paper describes some of the current methods of the deformation analysis. For the purpose of Deformation Analysis Methods application, a geodetic control network was conceptualized, with simulations of all deformations. By the application of methods by Pelzer, by Karlsruhe, by the modified Karlsruhe method and the one being implemented in the JAG3D, the Open-Source Software has identified unstable points giving movements of determined points. Points 6 and 7 were identified as unstable, while the remaining points were identified as stable. Nevertheless for other points no significant movements have been discovered, with simulated movements being on the measurement accuracy threshold. Information about stability, simulated and determined points' movements are shown in Table 8. Movements detected through application of the above Deformation Analysis Methods are nearly identical and comparable to the simulated movements. The greatest difference has the magnitude order of several millimeters, referring to the point 7. Reliability of the Deformation Analysis Methods is dependent on stable datum framework, along with the accuracy of measurement, indicating the lower threshold of movement that can be discovered "with certainty", which is verified by the results obtained in the practical part of this paper (points with simulated smaller movements were not identified as unstable).

Table 8: Simulated and identified movements of points.

Method	Points	Basic network points					Construction points			
		1	2	3	4	5	6	7	8	9
Simulated	*dy* [mm]	0	0	0	0	0	-9.21	-28.01	-3.03	-0.15
	dx [mm]	0	0	0	0	0	-11.84	-28.56	-9.53	-5
	d [mm]	0	0	0	0	0	15	40	10	5
	v [°]	--	--	--	--	--	218	225	197	182

Method	Points	Basic network points				Construction points				
		1	2	3	4	5	6	7	8	9
Pelzer	*dy* [mm]	0	0	0	0	0.01	-11.93	-28.11	-0.89	0.65
	dx [mm]	0	0	0	0	0.92	-7.39	-19.68	-5.42	0.46
	d [mm]	0	0	0	0	0.92	14.03	34.31	5.49	0.80
	ν[°]	--	--	--	--	0.62	238.22	235.00	189.33	54.71
	Stable	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**No**	**No**	**Yes**	**Yes**
Karlsruhe	*dy* [mm]	0	0	0	0	0.01	-11.93	-28.11	-0.89	0.65
	dx [mm]	0	0	0	0	0.92	-7.39	-19.68	-5.42	0.46
	d [mm]	0	0	0	0	0.92	14.03	34.31	5.49	0.80
	ν[°]	--	--	--	--	0.62	238.22	235.00	189.33	54.71
	Stable	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**No**	**No**	**Yes**	**Yes**
Modified Karlsruhe	*dy* [mm]	-0.27	0.47	-1.06	0.86	0.01	-11.93	-28.11	-0.89	0.65
	dx [mm]	-0.34	-1.62	2.79	-0.83	0.91	-7.41	-19.70	-5.44	0.43
	d [mm]	0.43	1.69	2.98	1.2	0.91	14.04	34.33	5.51	0.78
	ν[°]	218.45	163.82	339.20	133.98	0.63	238.15	234.98	189.29	56.51
	Stable	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**No**	**No**	**Yes**	**Yes**
JAG3D	*dy* [mm]	0	0	0	0	0.01	-11.92	-28.11	-0.89	0.65
	dx [mm]	0	0	0	0	0.92	-7.39	-19.68	-5.41	0.46
	d [mm]	0	0	0	0	0.92	14.02	34.31	5.48	0.80
	ν [°]	--	--	--	--	0.62	238.20	235.00	189.34	54.71
	Stable	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**No**	**No**	**Yes**	**Yes**

8 CONCLUSIONS

It is known that certain methods of deformation analysis provide different results – the conclusion being made by the FIG Commission, consisting of esteemed scientists from five university centers (Hanover, Delft, Fredericton, Karlsruhe and Munich). Of course, conclusions on reliability of Deformation Analysis Methods cannot be made based on the research performed on a single deformation model. Examining reliability of individual deformation analysis method needs to be implemented over several different deformation models.

Pursuant to the results obtained in the paper, the conclusion can be drawn that modified Karlsruhe Method and method implemented in the JAG3D Open-Source Software provide satisfying results regarding the discovery of unstable points, against the most commonly applied Deformation Analysis Methods–by Pelzer and Karlsruhe.

Precondition for the efficient application of various Deformation Analysis Methods refers to the setting as many basic points on a geologically stable terrain as possible. Apart from reliability of information on ground and construction movements, an important factor refers to the least movement intensity determination, which may certainly be determined with chosen probability and the test power.

If a minimum of four datum points are situated on geologically stable terrain, outside zone of expected deformations, having not changed their positions between two epochs of measurement, reliability of applying models noted in the paper will significantly increase. That is also impacted by application of the GNSS measurements using static or rapid static method, especially when significant movements in horizontal plane are being quantified.

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