RECENZIRANI ČLANKI | PEER-REVIEWED ARTICLES

TRANSFORMACIJA DEFORMIRANIH GEODETSKIH MREŽ V NOVE REFERENČNE KOORDINATNE SISTEME: ŠTUDIJA PRIMERA TRANSFORMACIJE ED50-ITRFXX V TURČIJI

TRANSFORMATION OF DISTORTED GEODETIC NETWORKS TO NEW COORDINATE REFERENCE SYSTEMS: A CASE STUDY FOR ED50-ITRFXX TRANSFORMATION IN TURKEY

Metin Soycan, Arzu Soycan, Nursu Tunalıoğlu

UDK: 528.2+528.5(560):
Klasifikacija prispevka po COBISS.SI: 1.01
Prispelo: 25. 10. 2016
Sprejeto: 16. 3 .2017

DOI: 10.15292/geodetski-vestnik.2017.01.58-75
SCIENTIFIC ARTICLE
Received: 25. 10. 2016
Accepted: 16. 3. 2017

SI | EN

IZVLEČEK

Transformacija med mednarodnim terestričnim referenčnim sestavom ITRFXX (angl. International Terrestrial Reference Frame), realiziranim za lokalne in regionalne geodetske mreže, ter geodetskim datumom ED50 (angl. European Datum 1950) se v praksi izvaja z več transformacijskimi modeli. Včasih modeli, ki se uporabljajo v praksi, niso primerni za nekatere transformacije in lahko občutno zmanjšujejo točnost transformiranih koordinat točk. V študiji smo najprej obravnavali polinomsko transformacijo s splošnimi enačbami in izraženo s kompleksnimi števili kot alternativno rešitev za dobro poznane transformacijske modele za modeliranje distorzij. Dodatno smo obravnavali transformacijske modele, ki temeljijo na radialnih baznih aktivacijskih funkcijah in predstavljajo sodoben pristop k ocenjevanju funkcij z več spremenljivkami. Transformacijski problem je prikazan na dveh numeričnih študijskih primerih, ki so stvarni podatki iz dveh regij v Turčiji.

ABSTRACT

The transformations between ITRFXX (International Terrestrial Reference Frame) established for local and regional geodetic networks and ED50 (European Datum 1950) are routinely implemented in practice via several transformation models. In some specific cases, these models, which are widely used in practice, may be insufficient to solve the transformation problem, and thus this causes a significant loss of accuracy for the transformed points coordinates. In this study, firstly the polynomial transformations with general equations (GP) and complex numbers (CNP) were examined as alternative to well-known transformation models for modelling the distortions. In addition, transformation models based on radial basis functions (RBFs), a modern method for estimating multivariable functions, have been examined. The transformation problem has been addressed in two numerical case studies, with real data located in different regions of Turkey.

KLJUČNE BESEDE

transformacija koordinat, ED50, ITRF, splošna polinomska transformacija, transformacija s kompleksnimi števili, radialne bazne aktivacijske funkcije

KEY WORDS

coordinate transformation, ED50, ITRF, general polynomial transformation, transformation with complex numbers, radial basis functions

1 INTRODUCTION

With the development of the GNSS technology, the need for a globally valid terrestrial reference frame has become inevitable. A terrestrial reference frame provides a set of coordinates of some points located on the Earth's surface. This is an extremely important component in terms of geodetic studies that require high accuracy. The International Terrestrial Reference System (ITRS) is a world spatial reference system co-rotating with the Earth in its diurnal motion in space. The International Earth Rotation Service (IERS), in charge of providing global references to the astronomical, geodetic and geophysical communities, supervises the realization of the ITRS. Realizations of the ITRS are produced by the IERS ITRS Product Center (ITRS-PC) under the name International Terrestrial Reference Frames (ITRF) (URL-1). Similarly, European Terrestrial Reference System (ETRS89) is defined coincident with the ITRS at the epoch 1989.0 for Europe and moving with the stable part of the Eurasian Plate (Adam et al., 2000). Another reference system based on the ITRS or other geodetic coordinate reference systems compliant with the ITRS is required in the areas that are outside the geographical scope of the ETRS89 (URL-2).

On the other hand, ED50 (European Datum 1950) is a geodetic datum, which was defined after the World War II for the international connection of geodetic networks based on the international Hayford ellipsoid. This was an early attempt to model the whole Earth and was widely used around the world until the 1980s when GRS80 and ITRF were established. Thus, during this period, countries defined their national geodetic networks based on this system and continued their geodetic activities. Spatial information generated in those systems needs to be transformed nowadays with appropriate coordinate transformation models so that they can be adapted to the newly defined coordinate reference systems.

Due to the several reasons, such as survey techniques applied for the establishment of the traditional classical geodetic networks, parameters omitted on computations and differences in ellipsoidal size between ED50 and ITRFXX have caused geometrical problems in transformation. Additionally, geophysical phenomena such as tectonic movements, earthquakes, brought geophysical problems, which have been merged with the geometric problems. Here, a complex problem on transformation, known as distortion modellling, has been raised. In such a case, obtaining transformation parameters by well-known transformation models between distorted coordinates of the old national network and undistorted GNSS networks is not a very easy process at the desired level of accuracy, as noted by several authors (IGNA, 1999; Tokhey, 2000; Ayan et al., 2001; Kutoğlu et al., 2001; Soycan, 2005; Ayan et al., 2006; Soycan and Soycan, 2008; Soycan and Soycan, 2014). Alternative transformation models (multi-variational approach) mentioned in this study are generally used for distortion modelling, which occurs when the homogeneities of scaling and direction are missing between two coordinate systems. The two-dimensional coordinate offsets between two coordinate systems are modelled by representing the common point's positions with an appropriate function for multi-variational approach (Calvert, 1995; Fogel and Tinney, 1996; NIMA, 1997; Tokhey, 2000; Greaves and Cruddace, 2001; Greaves and Cruddace, 2002; Soycan, 2005; Mitas and Mitasova, 2005).

2 A BRIEF OVERVIEW OF TRANSFORMATION MODELS

The relationship between geodetic coordinate systems is theoretically provided by 2 or 3 dimensional well-known transformation models. Generally, the accuracies of ED50 coordinates determined in old networks are lower than GNSS-based new networks due to some restrictions of the surveys with terrestrial classical techniques. However, because of systematic and non-systematic bias and physical factors resulting in measurement and calculation errors, geodetic networks may be distorted. This is especially the case in the transformations around the fault lines, where the point locations change due to the tectonic movements. Although the position change is usually in the form of translation, it also causes a scale change on the edges of the fault. The difference between the flattening of the reference ellipsoids of ED50 and ITRFXX also causes a change in the scale depending on the latitude value. In single scale models, different scale effects are ignored. In such a case, these well-known and frequently used transformation models may not be sufficient. A different model should be chosen to solve this problem.

Several transformation models can be implemented to transform coordinates from one system to the target system. The transformation from ED50 to ITRFXX can be achieved by the approaches such as (OGP, 2006):

— 3D transformations with geocentric (X, Y, Z) coordinate (geocentric translations, Helmert 7-parameter, Molodensky-Badekas etc.),
— 2D transformations with projected (north, east) coordinates (similarity, affine, polynomial etc.),
— 2D or 3D transformation with ellipsoidal geographical (φ, λ, h) coordinates (Abridged Molodensky, geographic offsets modelling with interpolation methods etc.).

In practice, to carry out a 3D transformation, cartesian coordinates (X, Y, Z) of common points need to be known in both systems. It is possible to easily reach cartesian or geographical coordinates (φ, λ, h) of points as the results of the GNSS data processing. On the other hand, the calculations of ED50 networks have been done separately as horizontal and vertical networks densification approaches ordinarily. The ellipsoidal heights (h) of points within the ED50 are generally unknown and are obtained as the sum of the orthometric height (H) and the geoid height (N). In a 3D transformation model with incorrect knowledge of ellipsoidal height data, transformation coefficients are affected from incorrect ellipsoidal height data (Soycan, 2008). Hence, the ellipsoidal heights of the common points are ignored due to the reasons such as determining the cartesian coordinates (X, Y, Z) of common points in ED50 (Vaníček and Steeves, 1996). Two-dimensional transformation approaches with geographic offsets, which provide more effective solutions for modelling the distortions, may be considered.

General formulas used commonly in each model to be explained in subsections are mentioned in (1) for development of a two-dimensional transformation model with ellipsoidal geographical coordinates between two systems by using geographic offsets.

$$\Delta\varphi = (\varphi_E - \varphi_I) \cdot m; \Delta\lambda = (\lambda_E - \lambda_I) \cdot m; u = \varphi - \varphi_0; v = \lambda - \lambda_0 \quad (1)$$

Where, φ_E, λ_E and φ_I, λ_I are the geodetic coordinates of the common points in the source (ED50) and target (ITRFXX) dataset, respectively. m is the scale that is applied to the coordinate differences for reducing them into a numerical range and enables to implement into the polynomial formulae without introducing numerical precision errors (OGP, 2006).

2.1 Polynomial transformations

Polynomial transformation, also known as a multivariate regression in practice, is frequently preferred in terms of practicability, ease of calculation and applicability. In general, polynomials can be either orthogonal or non-orthogonal. They provide satisfactory solutions for 2nd and 3rd order polynomials. The least squares fitting method is used to calculate the transformation coefficients from common points. Depending upon the degree of the distortion, complex polynomial equations may be required. However, for higher order polynomial solutions, more common points are needed. Moreover, there will be geometrical problems at the border of the study area, where the point density is not sufficient. The potential problem is numerical instability for polynomial transformation. In the case of using projected coordinates, scaling is also needed. Thus, the polynomial function may be defined as given below in terms of geographic offset ($\Delta\varphi$ and $\Delta\lambda$). The general polynomial (GP) equations of $\Delta\varphi$ and $\Delta\lambda$ given in (2) and (3) can be written with a_i and b_i coefficients as following:

$$F(u, v) = \Delta\lambda = b_0 + b_1u_1 + b_2v_1 + b_3u_1v_1 + b_4u_1^2 + b_5v_1^2 + \cdots \tag{2}$$

$$G(u, v) = \Delta\varphi = a_0 + a_1u_1 + a_2v_1 + a_3u_1v_1 + a_4u_1^2 + a_5v_1^2 + \cdots \tag{3}$$

The matrix system of observation equations can be defined as;

$$\underline{L} = \underline{A}\,\underline{X} - \underline{e} = \begin{bmatrix} \Delta_{\lambda 1} & \Delta_{\varphi 1} \\ \Delta_{\lambda 2} & \Delta_{\varphi 2} \\ \cdots & \cdots \\ \cdots & \cdots \\ \Delta_{\lambda \mathrm{n}} & \Delta_{\varphi \mathrm{n}} \end{bmatrix} = \begin{bmatrix} 1 & u_1 & v_1 & u_1v_1 & u_1^2 & v_1^2 & \cdots \\ 1 & u_2 & v_2 & u_2v_2 & u_2^2 & v_2^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & u_n & v_n & u_nv_n & u_n^2 & v_n^2 & \cdots \end{bmatrix} \begin{bmatrix} a_0 & b_0 \\ a_1 & b_1 \\ a_2 & b_2 \\ \cdots & \cdots \\ a_n & b_n \end{bmatrix} - \begin{bmatrix} e_{\lambda 1} & e_{\varphi 1} \\ e_{\lambda 2} & e_{\varphi 2} \\ \cdots & \cdots \\ \cdots & \cdots \\ e_{\lambda n} & e_{\varphi n} \end{bmatrix} \tag{4}$$

As another option for transformation with GP, polynomial transformation with complex numbers (CNP), which can be found in literature may be implemented. The CNP, which is one of the methods that can be used as an alternative, gives quite appropriate results in practice. It is a method that comes to the forefront especially by estimating a fewer number of high-order polynomial coefficients meaningfully. The relationship between two coordinate systems is designed more regularly with a single polynomial function defined by complex numbers and the transformation coefficients can be estimated. Thus, it is ensured that the dependency between separately estimated polynomial coefficients for both axes in conventional polynomial transformation can be solved with fewer coefficients than conventional polynomial transformation with the same order polynomial by single equation. The transformation equation can be defined with $\Delta\varphi$ and $\Delta\lambda$ as given with c_j coefficients in (5) for complex numbers.

$$F(u, v) = (\Delta\lambda + i.\Delta\varphi) = (c_1 + i.c_2)(u + i.v) + (c_3 + i.c_4)(u + i.v)^2 + (c_5 + i.c_6)(u + i.v)^3 + \cdots \tag{5}$$

The matrix system of observation equations can be defined as:

$$\underline{L} = \underline{A}\,\underline{X} - \underline{e} = \begin{bmatrix} \Delta_{\lambda 1} \\ \Delta_{\varphi 1} \\ \Delta_{\lambda 2} \\ \Delta_{\varphi 2} \\ \cdots \\ \cdots \\ \Delta_{\lambda n} \\ \Delta_{\varphi n} \end{bmatrix} = \begin{bmatrix} u_1 & -v_1 & (u_1^2 - v_1^2) & -2u_1v_1 & (u_1^3 - 3u_1v_1^2 - v_1^2) & -(3u_1^2 - v_1^3) & \cdots \\ v_1 & u_1 & 2u_1v_1 & (u_1^2 - v_1^2) & (3u_1^2 - v_1^3) & (u_1^3 - 3u_1v_1^2 - v_1^2) & \cdots \\ u_2 & -v_2 & (u_2^2 - v_2^2) & -2u_2v_2 & (u_2^3 - 3u_2v_2^2 - v_2^2) & -(3u_2^2 - v_2^3) & \cdots \\ v_2 & u_2 & 2u_2v_2 & (u_2^2 - v_2^2) & (3u_2^2 - v_2^3) & (u_2^3 - 3u_2v_2^2 - v_2^2) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ u_n & -v_n & (u_n^2 - v_n^2) & -2u_nv_n & (u_n^3 - 3u_nv_n^2 - v_n^2) & -(3u_n^2 - v_n^3) & \cdots \\ v_n & u_n & 2u_nv_n & (u_n^2 - v_n^2) & (3u_n^2 - v_n^3) & (u_n^3 - 3u_nv_n^2 - v_n^2) & \cdots \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ \cdots \\ \cdots \\ c_{n-1} \\ c_n \end{bmatrix} - \begin{bmatrix} e_{\lambda n} \\ e_{\varphi 1} \\ e_{\lambda n} \\ e_{\varphi 2} \\ \cdots \\ \cdots \\ e_{\lambda n} \\ e_{\varphi n} \end{bmatrix} \tag{6}$$

Although the GP coefficients are reversible, the reversibility of the CNP is not possible. For this method, the transformation coefficients should be computed using the same formulation in reverse transformation (OGP, 2006; Zeng, 2014; Ruffhead, 2017).

2.2 Multi-variational approach (transformation with RBF)

The RBF is a modern estimation method used for estimation of multi-variation functions (Mitas and Mitasova, 2005), so called variational approach. The variational approach offers a wide range of possibilities to incorporate additional conditions such as value constraints, prescribed derivatives at the given or at arbitrary points, and integral constraints (Talmi and Gilat, 1977; Wahba, 1990; Fogel and Tinney, 1996; Schaback, 2007).

Estimation is achieved by using several types of functions depending on the distances between control points (common points in both systems). They are often used for solutions of the interpolation problems generated by the irregularly distributed dataset. Several functions may be defined as RBFs, which are scalar functions whose values are only dependent on the distance from the origin of the point where the function is calculated as below with the m_i and k_i coefficients. The surface spline as described by Goshtasby (1988) and Flusser (1992) is shown below for $\Delta\varphi = F(u, v)$ and $y = \Delta\lambda(u, v)$;

$$F\left(u,v\right) = \Delta\lambda = T\left(u,v\right) + \Sigma_{i=1}^{n} k_i . R\left(r_j\right) = f_0 + f_1 u + f_2 v + \Sigma_{i=1}^{n} k_i Q_i \tag{7}$$

$$G\left(u,v\right) = \Delta\varphi = T\left(u,v\right) + \Sigma_{i=1}^{n} m_i . R\left(r_j\right) = d_0 + d_1 u + d_2 v + \Sigma_{i=1}^{n} m_i Q_i \tag{8}$$

where r_i is the distance from the point to the i^{th} point, $T(r)$ is the trend function (a constant term or the first order polynomial can be used as depending on function type and constraints), $R(r, r_i)$ is RBF to use to determine the Q_i weighting coefficients. The interpolation function approaches to zero if the distance between the common point and the estimated point increases. As a result, the weights are getting higher if the points are close to the point to be estimated. Similarly, if the points are far away from the points to be estimated, the weights will be lower. In order to have square integrable second derivatives, the additional conditions of polynomial terms should be as follow. The following "equilibrium constraints" are imposed:

$$\Sigma_{i=1}^{n} k_i = \Sigma_{i=1}^{n} k_i \Delta\varphi_i = \Sigma_{i=1}^{n} k_i \Delta\lambda_i = 0 \text{ and } \Sigma_{i=1}^{n} m_i = \Sigma_{i=1}^{n} m_i \Delta\varphi_i = \Sigma_{i=1}^{n} m_i \Delta\lambda_i = 0 \tag{9}$$

The matrix system of observation equations can be defined as:

$$\underline{L} = \underline{A}\,\underline{X} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \Delta_{\lambda 1} & \Delta_{\varphi 1} \\ \Delta_{\lambda 2} & \Delta_{\varphi 2} \\ \cdots & \cdots \\ \Delta_{\lambda n} & \Delta_{\varphi n} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 0 & u_1 & u_2 & \cdots & u_n \\ 0 & 0 & 0 & v_1 & v_2 & \cdots & v_n \\ 1 & u_1 & v_1 & 0 & Q_{12} & \cdots & Q_{1n} \\ 1 & u_2 & v_2 & Q_{21} & 0 & \cdots & Q_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & u_n & v_n & Q_{n1} & Q_{n2} & \cdots & 0 \end{bmatrix} \begin{bmatrix} d_0 & f_0 \\ d_1 & f_1 \\ d_2 & f_2 \\ m_1 & k_1 \\ m_2 & k_2 \\ \cdots & \cdots \\ m_n & k_n \end{bmatrix} \tag{10}$$

2.3 Quantitative measures for accuracy of the transformation models

The unknown coefficients of the models with their covariance information are simply determined according to the Least Squares Adjustment (LSA) principles, which are minimizing the sum of squares of the residuals e_i, as follow with the equal weights.

$$\underline{X} = \left(\underline{A}^T \underline{A}\right)^{-1} \left(\underline{A}^T \underline{L}\right); \ \underline{e} = \underline{A}\,\underline{X} - \underline{L} \tag{11}$$

$$\sigma_0^2 = \frac{v^T v}{2q - r}; \ \sum \hat{x}\hat{x} = \hat{\sigma}_0^2 \left(A^T A\right)^{-1} \tag{12}$$

Here, $\underline{L}$ is the observation vector, which consists of geographic offsets between two systems, $\underline{X}$ is the estimated value of the unknown transformation coefficients vector, $\underline{A}$ is the design matrix, $\underline{e}$ is the residual vector.

$\underline{X}$ includes transformation coefficients, namely a_i, b_i, c_i,… m_i, k_i, corresponding to GP, CNP and RBFs, respectively. σ_0^2 is the variance of unit weight, $\Sigma\hat{x}\hat{x}$ is covariance matrix of $\underline{X}$ vector, q is the number of common points, r is the unknown parameter number (number of transformation coefficients).

If the transformation coefficients are used to transform the common points, the transformed values will not match with their true values and there will be a difference called a residual. After the transformation coefficients with the least squares are derived, the residuals can be calculated by applying the transformation coefficients to all common points (11). The inverse of the residual is defined as error and represents the difference between the actual position values of the points and the values calculated by the transformation coefficients. The square root of the variance of unit weight (σ_0) computed from residuals is traditionally considered as the internal accuracy of the model and this is an important value for statistical tests (i.e. significance test for coefficients, outlier detection etc.) to be applied. Although σ_0 is a good assessment of the transformation's accuracy, it cannot be concluded that a lower values of σ_0 yields an accurate transformation. Some transformation models give residuals nearly zero or zero; because the transformation surface passes through the given common points, so these points have no residuals (i. e. in equation (10); number of equations is equal number of unknowns). This does not mean that the coordinates will be perfectly transformed without errors. The transformation may still contain significant errors.

Generally, cross validation statistic can be considered as a measure of the transformation error for assessing the quality of the models for this case. These are very important indicators to evaluate the appropriateness of the transformation models. Thus, to analyse the models statistically, "cross-validation" process could be applied to the common points. In this process, one of the common points is removed from the dataset, and the rest of the common points are used to estimate coefficients. With the help of coefficients calculated, the common point removed from the dataset is estimated. In the next stage, this point is added into the dataset again, and the same process is repeated to the other common points one by one. After applying this process to all common points, estimation errors (ε) can be obtained from the differences between estimated and actual values for latitude and longitude components. Besides, cross validation results are considered as an important indicator to identify the outlier detection and provide information on the spatial distribution of the data as well.

Consistency with the transformation model, each common point can be interpreted by the estimation error magnitudes and the several statistical measures can be derived from them. In this sense, statistical information such as; mean error (ME), root mean square (RMS) error, standard error (SE), standard deviation (STD), median absolute deviation (MAD) etc. can be used for understanding the model quality. Among the statistical measures, the most significant value, which gives an idea about the accuracies of the transformation models, is the RMS of estimation error. The RMS is calculated by summing the mean squares of the errors for latitude and longitude components (13a–13b). Then, the total RMS can be calculated by (13c). RMS indicates how closely model estimates the measured values. The smaller this error, the better estimation could be performed.

$$RMS_{\varphi} = \sqrt{\Sigma_{i=1}^{n}\left(\varepsilon_{\varphi}\right)^{2} / n} \tag{13.a}$$

$$RMS_{\lambda} = \sqrt{\Sigma_{i=1}^{n}\left(\varepsilon_{\lambda}\right)^{2} / n} \tag{13.b}$$

$$RMS_{Total} = \sqrt{\Sigma_{i=1}^{n}\left(\varepsilon_{\varphi}\right)^{2} + \left(\varepsilon_{\lambda}\right)^{2} / n} \tag{13.c}$$

With cross-validation, the following results are expected;

- the average of errors close to zero;
- a small RMS error for prediction;
- a standardized mean prediction error near zero;
- an average standard error similar to the RMS.

However, RMSE values obtained by cross validation are larger than σ_0 values obtained by the residuals of least squares solutions. One can achieve more representative results due to the fact that the error values at each point are calculated out of the data set every time, which better represents the real situation.

3 A CASE STUDY FOR ED50-ITRFXX TRANSFORMATION IN TURKEY

Turkey is the country affected by several different faults such as the Black Sea plate, Eurasian plate, Aegean plate, African plate, Arabian plate, and Anatolian plate (Figure 1). Due to this structure, almost 92% of the country area is under the risk of earthquake. Most of the micro geodetic networks have been established for monitoring geodynamic activities on the North Anatolian fault (NAF) (Milev et al., 2010). In Turkey, which is located in the zone of convergence between the Africa, Arabia, and Eurasia plates (McKenzie, 1976), points coordinates have been shifted with time due to deformations and seismicity occurred by tectonics.

Turkish National Fundamental GPS Network (TUTGA) has been established between 1997 and 1999 and some of the stations have been resurveyed due to the earthquakes, which happened in 1999 (Reilinger et al., 2000; Bürgmann et al., 2002). The total number of stations is 596, each station with known 3D coordinates and their associated velocities have been computed. Turkish National Reference Frame is called TUREF, which was derived from ITRF96 depending on Turkish National Fundamental GPS Network (TUTGA-99A, 2002). TUREF was defined to supply a reference frame, which might be independent of future versions of ITRS. In this context, the new reference frame to be defined for maintaining applications of geodetic infrastructures of Turkey and large scale mapping facilities coordinately, should be coincident with ITRF96. Accordingly, TUREF was defined as to coincide with ITRF96 at the epoch 2005.0 due to the reasons that both arise from geodynamic properties of Turkey and constraints related with geodetic infrastructure/coordinate reference system conducted for a long time for geodetic tasks (Aktuğ et al., 2010; Aktuğ et al., 2011).

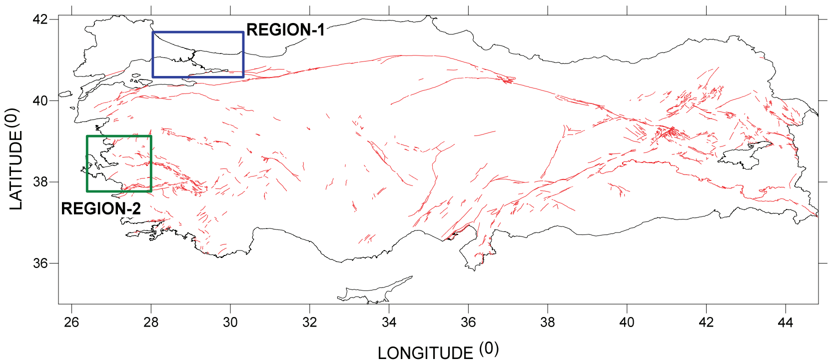
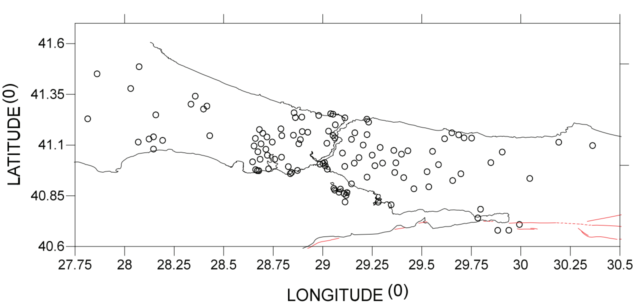


Figure 1: Active fault lines and locations of both study regions.

The nation-wide studies should be done for adaptation of ETRS89, which is routinely used in several European countries. Thus, transformation parameters between TUREF and ETRS89 were calculated (URL-3) and several studies have been conducted to improve the reference frame. Moreover, strategy for updating the reference epoch is still being studied.

3.1 Test data and test regions

The experimental study was performed using two datasets. The first data was selected from the project of Istanbul GPS Network updated 2005–2006 surveys and the second data was selected from the project of Izmir geodetic infrastructure for the production of 1/5000 scaled digital photogrammetric maps and orthophotos (Alkış et al, 2011).



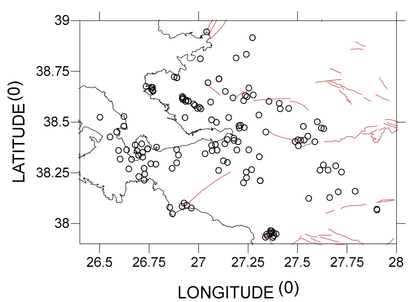


Figure 2: Study regions and common points used in transformation.

Figure 1 represents active fault lines with directions as indicated with red lines. To review the transformation models applied in this study, the study regions were selected depending on fault lines. Here, Figure 2 shows the study regions located in İstanbul (Region-1, Figure 2 left) and in İzmir (Region-2, Figure 2 right).

Region-1 involves 115 common points with known coordinates. Geodetic coordinates of the common points to be used for transformation are in both national coordinate system ED50 and ITRF96 datum, epoch 2005. The region covers approximately 30 km × 90 km, which is located at latitudes between 40.68°, and 41.49° and longitudes between 27.81° and 30.36° in ITRF96 datum. The geographic offsets from ED50 to ITRF96 in the area of interest vary approximately from 3.42 second (~106 m) to 3.51 second (~108 m) in latitude direction and from 1.69 second (~44 m) to 1.43 second (~52 m) in longitude direction, respectively. For Region1, as the relation between ED50 and ITRF96 latitudes and longitudes differences of common points and latitudes-longitudes values of points was examined, it has been observed that there are high correlations with correlation values 0.9771 and 0.9824, between latitude differences to latitude values and longitude differences to longitude values, respectively. Furthermore, it has been shown that there are important correlations with correlation values 0.5620 and 0.5166, between latitude differences to longitude values and longitude differences to latitude values, respectively.

Region-2 involves 208 common points with known coordinates and 145 of them were used in this study. Geodetic coordinates of common points to be used for transformation are in national coordinate system ED50 and ITRF2005 datum, epoch 2005. The region covers approximately 115 km × 112 km, which is located at latitudes between 37.87° and 38.91°, and longitudes between 26.47° and 27.76° in ITRF2005 datum. The geographic offsets from ED50 to ITRF96 on the area of interest vary approximately from 3.72 second (~115 m) to 3.82 second (~118 m) in latitude direction and from 1.60 second (~49 m) to 1.76 second (~55 m) in longitude direction, respectively. For Region-2, considering the relationship between ED50 and ITRF96 latitudes and longitudes differences of common points and individually latitudes-longitudes values of points, it has been observed that there are high correlations with correlation values 0.9537 and 0.9630, between latitude differences to latitude values and longitude differences to longitude values, respectively. The correlation between latitude differences to longitude is 0.4509, which is quite important. However, the correlation between longitude differences to latitude is 0.0592, which is weak.

In this study, the differences of latitudes and longitudes (offsets) have been multiplied by an appropriate scale factor, and then the observation vector is obtained. On the other hand, if the normalized coordinates of u and v are too large (30°–40°) or too small (1°–2°), this will cause condition defects on coefficients of normal equations and thus the accuracy of estimation will be affected. For this purposes, u and v are obtained by shifting the latitude and longitude to the center of gravity of transformation area.

Data used in the study (common points with known coordinates) can be understood better, when considering the following items coordinate transformation problems:

- Although ITRF coordinates are defined by XYZ cartesian coordinates, ED50 coordinates are based on the Turkish National Horizontal Control network and expressed in the projection system.
- The accuracy of ITRF coordinates of the common points are 2–3 cm in the latitude and longitude, 3–5 cm in the vertical direction. Although there is no clear data on the accuracy of ED50 coordinates, it can be said that it is less accurate than ITRF coordinates.
- According to the velocity field defined in TUTGA, the velocity vectors of the points in the test regions vary from 0.5 cm/year to 3 cm/year.
- For transformation, the coordinates of the two systems are compiled from different public insti-

tutions. ED50 and ITRFXX coordinates of the common points have been generated for different projects and studies. Coordinates of common points in ED50 were calculated based on classical terrestrial observations and some of them contain different types of systematic errors.

— The vast majority of the points in question have only horizontal coordinates. On the other hand, the heights of significant sections of the upper grade points of the Horizontal Control Network are determined by the trigonometric method. Even if the heights are determined by the geometric levelling, the accuracy of ellipsoidal height is less accurate than the latitude and longitude components due to the accuracy of geoid heights in ED50.

— Since corrections and reductions for geoid, plumb line etc. had not been applied to the measurements of the old classical network defined in ED50, there are systematic effects on the points used in the datum definition. Beside position changes depending on regional and local deformations and earthquakes and crustal movements, these factors significantly distorted the old classical network.

When considering these factors, the transformation between two systems using routinely applied transformation models is difficult.

3.2 Evaluation of the examined transformation models

In the study, initially, transformation problems have been solved by the two-dimensional similarity transformation model. When the standard deviations of the transformation parameters and residuals of observations are analysed, it is seen that the model has not achieved the expected accuracy. Residuals applied to the common points after transformation have not reached acceptable tolerance value for the first dataset through two directions and for the second dataset, especially through longitude direction. It is clear that points (except for center points) have big residuals and behave as outliers.

The first model considered is the Affine parametric transformation. In general, Affine transformation is done geometrically, which includes six transformation parameters; two translations, two rotation components and two scale factors. Moreover, Affine transformation can also be provided by parametric transformation, whose coefficients are computed by 1st order polynomial function. Here, parametric transformation has been implemented. Affine transformation has not provided any significant improvements as to initially implemented transformation models on results. The results are:

— For Region-1, the total RMS and σ_0 estimated from Affine transformation are 0.334 m and 0.175 m, respectively. The estimation errors for latitudes of points (ε_φ) range from –0.214 m to 0.436 m and the standard deviation is approximately 0.102 m. The ε_φ for 14 of 115 common points (12 %) have exceeded 14 cm, which is the tolerable limit regulated by Turkish Large Scale Map and Map Information Production (Deniz et al., 2008). The estimation errors for longitudes of points (ε_λ) range from –0.515 m to 0.998 m; and the standard deviation is 0.318 m. The ε_λ for 79 of 115 common points (69 %) have exceeded 14 cm (see Figure 3).

— For Region-2, the total RMS and σ_0 estimated from Affine transformation are 0.258 m and 0.124 m, respectively. The ε_φ range from –0.422 m to 0.424 m, and the standard deviation is approximately 0.175 m. The ε_φ for 67 of 145 common points (46 %) have exceeded 14 cm. The ε_λ range from –0.347 m to 0.857 m; and the standard deviation is 0.189 m. The ε_λ for 51 of 145 common points (35 %) have been exceeding 14 cm (see Figure 3).

According to Figure 3, in Region-1, the estimation errors are greater in the east and south-east directions. In Region-2, this situation is observed in the south-east and north-west directions. According to the results, it is clear that distortion effects cannot be removed by Affine transformation either. In the case of insisting on a solution with these methods, most of the common points will be determined as outliers and should be removed from the dataset. Then the validation of transformation coefficients will decrease or unwanted situations will be raised, such as determining a point as an outlier, which creates geometrical problem if removed from transformation.

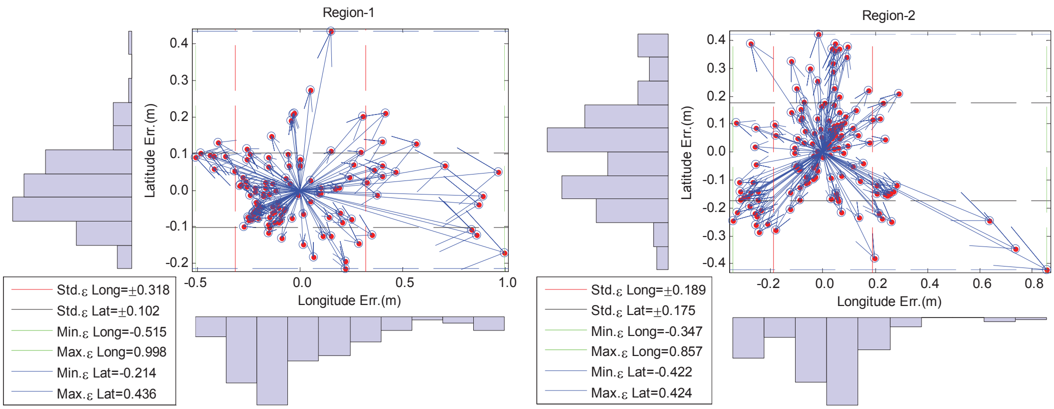


Figure 3: Scattered histograms for estimation errors for the Affine transformation.

Due to the problems on transformation with the Affine parametric model, the GP and CNP have been implemented to reduce the estimation errors. Here, the second model used is the 3rd order GP function and the third model is the 3rd order CNP. In both of GP and CNP, from 2nd to 6th order polynomials were implemented and the significance tests for transformation coefficients was applied. However, no significant changes in transformation coefficients for higher orders than 3rd order functions have been obtained. Therefore, 3rd order polynomial functions are used in both methods.

The most limitation factors for CNP can be listed as scaling the differences of latitudes and longitudes at the model and possible condition defects on the coefficients at normal equations. Evaluations on 3rd order GP transformation method may be summarized as follows:

— For Region-1, the total RMS and σ_0 obtained after polynomial transformation are 0.152 m and 0.105 m, respectively. The ε_φ range from –0.191 m to 0.226 m, and the standard deviation is approximately 0.080 m. The ε_λ range from –0.480 m to 0.433 m and the standard deviation is 0.129 m (see Figure 4). The number of common points exceeded 14 cm for ε_φ is 13 (11 %); the number of common points exceeded 14 cm for ε_λ is 22 (19 %).

— For Region-2, the total RMS and σ_0 obtained after polynomial transformation are 0.150 m and 0.091 m, respectively. The ε_φ range from –0.222 m to 0.252 m and the standard deviation is approximately 0.090 m. The ε_λ range from –0.364 m to 0.449 m and the standard deviation is 0.120 m (see Figure 4). The number of common points exceeded 14 cm for ε_φ is 22 (15 %); the number of common points exceeded 14 cm for ε_λ is 30 (21 %).

In Region-1 according to Figure 4, the estimation errors are greater in south-east and south-west directions. In Region-2, this situation is seen in south-east, north-east and west directions. As a clear statement provided from these results, distortion effect has been decreased significantly by 3rd order polynomial transformation when comparing with the Affine results (note that the axis scales are different in figures). According to the comparison for Region-1 and Region-2, the improvement in the standard deviations of ε_φ and ε_λ are 22% and 51%; 49% and 37%, respectively.

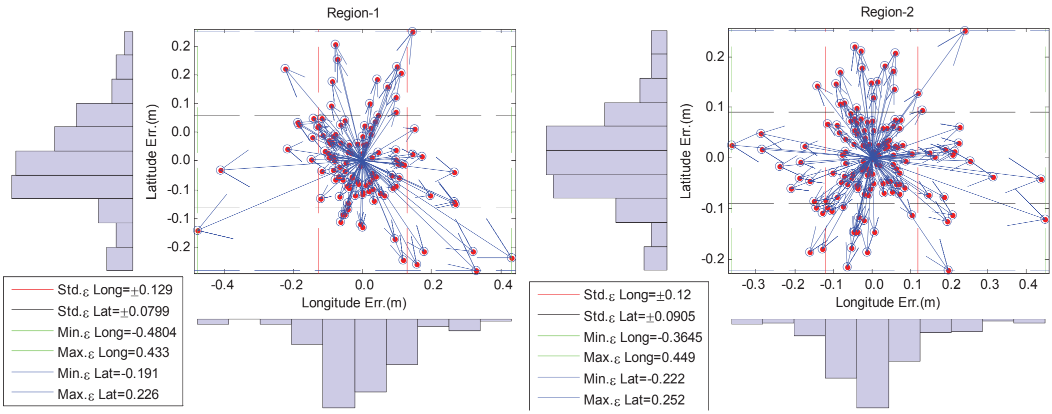


Figure 4: Scattered histograms for estimation errors for the 3rd order GP transformation.

On the other hand, transformation can be achieved easily with 6 coefficients calculated from 3rd order CNP. The results can be summarized as below:

— For Region-1, the total RMS and σ_0 are 0.162 m and 0.114 m, respectively. The ε_φ range from –0.202 m to 0.208 m and the standard deviation is 0.079 m. The ε_λ range from –0.431 m to 0.363 m; and the standard deviation is 0.142 m (see Figure 5). The number of common points exceeded 14 cm for ε_φ is 11 (10 %); the number of common points exceeded 14 cm for ε_λ is 30 (26 %).
— For Region-2, the total RMS and σ_0 are 0.173 m and 0.107 m, respectively. The ε_φ range from –0.263 m to 0.243 m and the standard deviation is 0.102 m. The ε_λ range from –0.355 m to 0.587 m, and the standard deviation is 0.140 m (see Figure 5). The number of common points exceeded 14 cm for ε_φ is 25 (17 %); the number of common points exceeded 14 cm for ε_λ is 33 (23 %).

When the results of CNP are considered, it can be exposed that this method gives more appropriate results than similarity, Affine and low-order GP transformation models. Residuals for coordinates of common points and standard deviation of unit weight obtained are lower than results of similarity and Affine transformation models, and very close to 3rd order GP transformation. Comparing the results with Affine for Region-1 and Region-2, the improvement in standard deviations for ε_φ and ε_λ are 23% and 55%; 42% and 26%, respectively.

As concluded from the above outcomes, transformation data sets still tend to lack of homogeneity of scale factor and exhibit local variations that are geographic offsets.

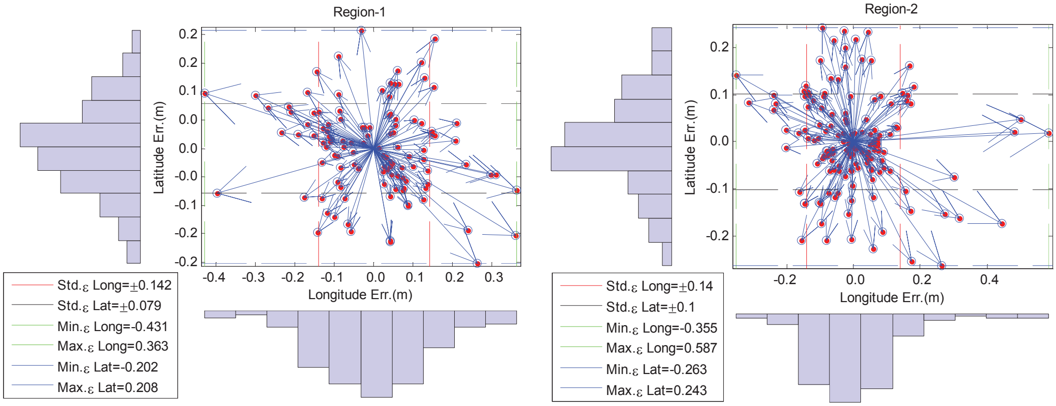


Figure 5: Scattered histograms for estimation errors for the CNP transformation.

In the final approach, we evaluate the RBFs and compare the transformation on the basis of three different basic functions. In this section, the multiquadric method (MQ), which is considered to be the easiest and originally developed version of RBF (Hardy, 1990), is used first. Besides, it has been evaluated with exponential (ES) and regularized spline (RS) functions. The basic idea is to choose a radially symmetric function and their additional parameters with optimized parameters by cross-validation. Thus, our main goal is to show how useful these models are in applications, in particular for modelling the locally variated significant estimation errors. The basis function can be defined for MQ, ES and RS models as given below:

$$R(r)=\sqrt{r^2+p^2} \tag{14}$$

$$R(r)=e^{-\sqrt{r^2+p^2}} \tag{15}$$

$$R(r)=\frac{1}{2\pi}\left(\frac{r^2}{4}\left(\ln\left(\frac{r}{2\tau}\right)+c-1\right)+\tau^2\left(K_o\left(\frac{r}{\tau}\right)+c+\ln\left(\frac{r}{2\pi}\right)\right)\right) \tag{16}$$

Where, τ^2 is the weight parameter, K_0 is the modified Bessel function and $c = 0.577215$ is the Euler constant for RS model. p^2 is the shaping factor specified by the user for MQs and ES model.

Although the RBF procedure given in Section 2.2 provides a straightforward way to obtain smooth and precise interpolations, it can be said that the choice of basic functions is arbitrary. In fact, the basic functions define the best set of weights to be applied to data points as it adds a point to the model. The problem is relatively simple, but it can only be solved with very sophisticated mathematical methods as to other transformation models. In general, the solvability of such a system is a serious question. This problem is solved by the generalized RBFs by the developing computing technologies and computer facilities. As with many software programs developed for this purpose, the problem can also be programming on different platforms. The results of the RBF transformation model:

— For Region-1, the total RMSs for ES, MQ and RS functions are 0.094 m, 0.079 m, 0.087 m, respectively. The ε_φ range from –0.157 m to 0.248 m and the standard deviations of ε_φ range from 0.043 m to 0.053 m. The ε_λ range from –0.381 m to 0.382 m the standard deviations of ε_λ range from 0.066 m to 0.078 m (see Figures 6–8).

— For Region-1, the total RMSs for ES, MQ and RS functions are 0.093 m, 0.097 m and 0.096 m, respectively. The ε_φ range from –0.186 m to 0.182 m and the standard deviations of ε_φ range from 0.064 m to 0.068 m. The ε_λ range from –0.164 m to 0.241 m the standard deviations of ε_λ range from 0.068 m to 0.072 m (see Figures 6–8).

In Figure 6–8, scattered pattern of estimation errors are still valid for solutions of RBFs. However, comparing with the other methods, they are very low. In Region-1, although the estimation errors are mostly centred, there are still scattered estimation errors. Similarly, in Region-2, the values are smaller than the other methods but not centred compared with Region-1. Here, distortion effect is decreased significantly and the standard deviations are improved. When comparing with CNP, improvement in standard deviations of ε_φ and ε_λ for ES, MQ and RS are 33% and 45%; 46% and 54%; 37% and 50% for Region-1 and Region-2 , respectively.

As the result of the RBF solutions, there is no significant difference between the different types of basic functions, on the other hand there seems to be a striking difference between RBF and the previous three transformation methods.

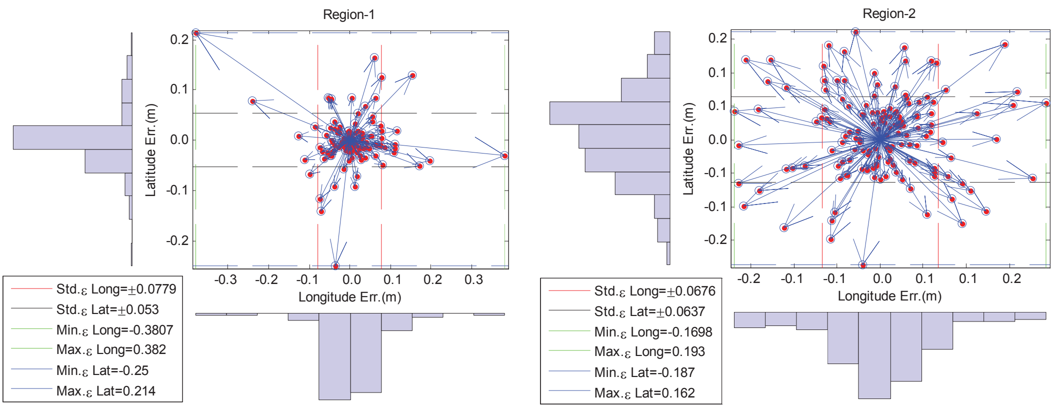


Figure 6: Scattered histograms for estimation errors for the ES transformation.

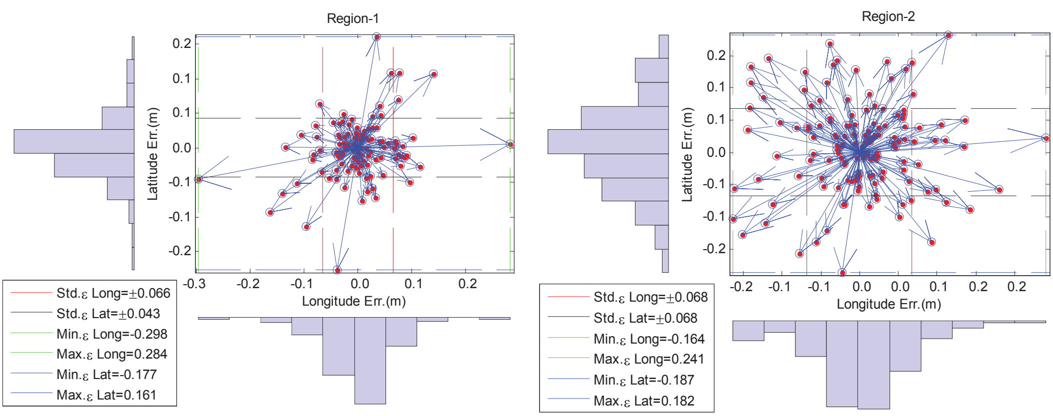


Figure 7: Scattered histograms for estimation errors for the MQ transformation.

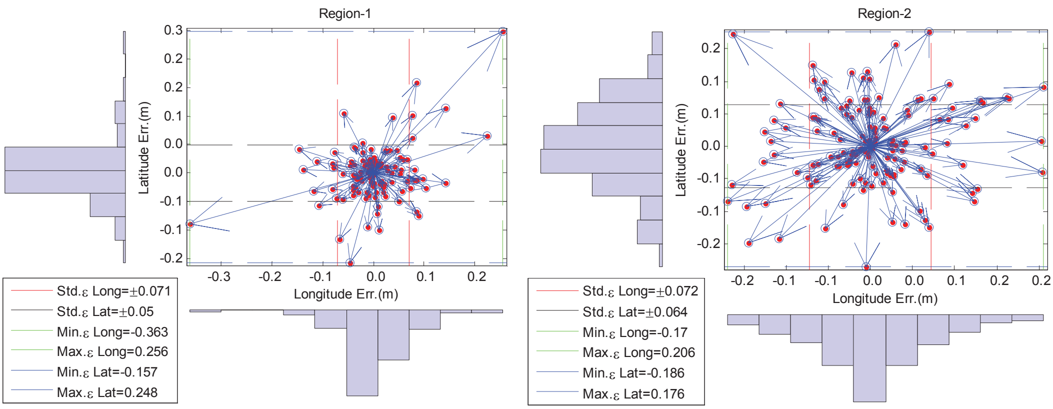


Figure 8: Scattered histograms for estimation errors for the RS transformation.

4 CONCLUSION

The Figure 9 is illustrating the Root Mean Square Errors for each transformation model used in this study for latitudes and longitudes.

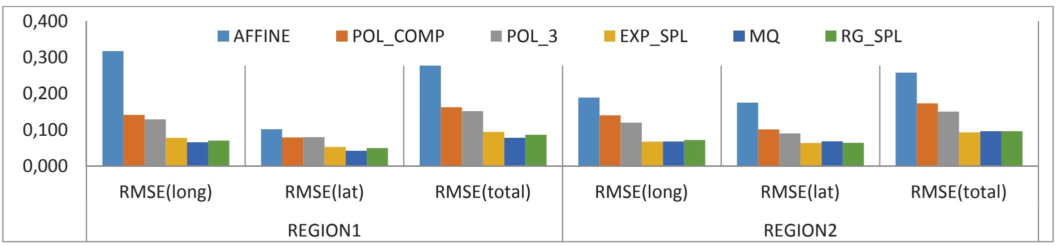


Figure 9: Root Mean Square Errors for each transformation model used in the study.

For the Region-1, the accuracies of the models, namely Affine transformation, CNP, 3rd order GP, ES, MQ and RS function were determined as 0.334 m, 0.162 m, 0.152 m, 0.094 m, 0.079 m and 0.087 m, respectively.

For the Region-2, the accuracies of the models namely Affine transformation, CNP, 3rd order GP, ES, MQ and RS function were determined as 0.258 m, 0.173 m, 0.150 m, 0.093, 0.096 m and 0.096 m, respectively.

We can conclude from the results of the study:

— GP transformation generates satisfactory solutions in terms of some factors such as being practical, useful and easy modelling structure. The higher the transformation order, the more complex the distortion that can be corrected. However, high order polynomials may behave differently if the densities of common points are low. Therefore, it may cause undesired strain or it can tighten at the edges of the transformation area. In this view, the GP transformation is more effective if distortions are small and have a low frequency. The transformation, since modelling high frequently distortions with polynomial expansion is more complex, does not provide satisfactory solutions. Then, the lower order polynomials tend to give a random type error, while the higher order polynomials tend to give an extrapolation error.

- The CNP generates solution with fewer coefficients than GP transformation. Therefore, this method may be chosen if the number of common points is insufficient. Here, it is not necessary to consider the differences of latitudes and longitudes separately like the other methods because transformation can be achieved with one equation. During the modelling process, due to the differences in coordinates between 4^{th}, 5^{th} or higher order functions in coefficient matrix, condition defects have occurred on coefficients in normal equations.
- Due to the use of a function depending on the distances between common points when modelling the differences of latitude and longitudes, RBF, which is highly effective and available for all data types and eliminates systematic errors, the method can be used even for modelling of high-frequency distortion.
- In this study, although transformation with RBF achieves the most proper results for both dataset, the solution process is complex and tough. Direct use of transformation coefficient estimated is not as easy as the other methods. However, the geographic offsets can be stored in grid data format and can be used easily in this form.
- In such a case, where the distortion is very low or does not exist, the RBF model causes meaningless strains and does not enhance the transformation accuracy, even disturbs it.
- Therefore, it is not always easy to create RBFs that guarantee good stability and small errors at all times and in all conditions. Besides, it is not guaranteed that the points far away from the control points and outside the transformation area are correct.
- RBF may not provide appropriate results when big geographic offsets occur in short distances and/or if the sample data contains outliers. In this regard, the outliers must be removed from the data set by appropriate methods before processing the data. Actually, this is the common problem for all transformation models. All of them work better when the common points are correct and they are required to be isolated from outliers. The more accurately model can be achieved by using more common points with equal quality for each model.
- The size of the transformation area is also important according to the accuracy of the results to be obtained. In case of extending the size of transformation area the probability of global solution is limited for all methods. A global transformation simply means that all the common points are used to derive a single mathematical model. By implementing transformation models locally (local methods use subsets of the data) they generate more effective solutions than global approach in this case.

Literature and references:

Adam, J., Reuther, C.-D., Grasso, M., Torelli, L. (2000). Active fault kinematics and crustal stresses along the Ionian margin of the southeastern Sicily. Tectonophysics, 326 (3–4), 217–239.
DOI: http://dx.doi.org/10.1016/S0040-1951(00)00141-4

Aktuğ, B., Kılıçoğlu, A., Lenk, O. (2010). INSPIRE Direktifi Kapsamında Avrupa Yersel Referans Sistemi (ETRS-89) ve Türkiye Ulusal Referans Çerçevesinin İncelenmesi, Harita Dergisi, 143, 1-12.

Aktuğ, B., Seymen, S., Kurt, M., Parmaksız, E., Lenk, O., Sezer, S., Özdemir, S. (2011). ED-50 (European Datum-1950) İle TUREF (Türkiye Ulusal Referans Çerçevesi) Arasında Datum Dönüşümü, Harita Dergisi, 146, 8-17.

Alkis, A. (Project Manager) et al. (2011). Izmir Büyükşehir Belediyesi Sayısal Fotogrametrik Harita- Ortofoto ve Jeodezik Altyapı Projesi- Teknik Rapor (Technical Report), Yildiz Teknik Universitesi. (In Turkish).

Ayan, T., Özlüdemir, T., Akyılmaz, O., Arslan, E., Denli, H. H. (2001). Geodetic Network Densification in Istanbul-IGNA. 4th Turkish-German Joint Geodetic Days, Berlin. Vol. II, pp. 515–520.

Ayan, T., Deniz, R., Ersoy, A., Çelik, R. N., Denli, H., Akyılmaz, O., Özşamlı, C., Özlüdemir, M. T., Erol, S., Erol, B., Acar, M., Mercan, H., Tekdal, H. (2006). İstanbul GPS Nirengi Ağı (İGNA) 2005–2006 Yenileme Ölçü ve Değerlendirmesi. Teknik Rapor, Cilt. 2. 2006. ISBN: 978-975-561-301-7.

Bürgmann, R., Ayhan M.E., Fielding, E.J., Wright, T.J., McClusky, S., Aktug, B., Demir, C., Lenk, O., Turkezer, A. (2002). Deformation during the 12 November 1999, Düzce, Turkey Earthquake, from GPS and InSAR data, Bull. Seismol. Soc. Am., 92, 161-171.

Calvert, C. (1995). Ordnance Survey Policy and Procedures for WGS84 to National Grid (OSGB36 Datum) Transformations. Proc RIN 95, Personal Navigation, London: Royal Institute of Navigation.

Flusser, J. (1992). An adaptive method for image registration. Pattern Recognition, 25 (1), 45–54. DOI: http://dx.doi.org/10.1016/0031-3203(92)90005-4

Fogel, D. N., Tinney, L. R. (1996). Image Registration using Multiquadric Functions, the Finite Element Method, Bivariate Mapping Polynomials and Thin Plate Spline. National Center for Geographic Information and Analysis. Technical Report, pp. 96–101.

Goshtasby, A. (1988). Image registration by local approximation methods. Image and Vision Computing, 6 (4), 255–261. DOI: http://dx.doi.org/10.1016/0262-8856(88)90016-9

Greaves, M., Cruddace, P. (2001). The OS's new Coordinate Transformation for Great Britain – GPS to OSGB36 National Grid Transformation. Geomatics World, 10 (1), 34–36.

Greaves, M., Cruddace, P. (2002). The Adoption of ETRS89 as the National Mapping System for GB, via a Permanent GPS Network and Definitive Transformation. EUREF Publication No. 10.

Hardy, R. L. (1990). Theory and applications of the multiquadratic biharmonic method. Computers & Mathematics with Applications, 19 (8–9), 163–208. DOI: http://dx.doi.org/10.1016/0898-1221(90)90272-L

IGNA-Istanbul GPS Network Project. (1999). Technical Report. Istanbul: Istanbul Technical University, Geodesy Division.

Kutoğlu, S. H., Ayan, T., Akçin, H. (2001). Error Propagation in 3D Transformation between GPS and National Horizontal Control Networks. 4th Turkish-German Joint Geodetic Days, Berlin. Vol. II, pp. 597–606.

McKenzie, D. (1976). The east Anatolian fault: A major structure in eastern Turkey. Earth and Planetary Science Letters, 29 (1), 189–193. DOI: http://dx.doi.org/10.1016/0012-821X(76)90038-8

Milev, G., Becker, M., Vassileva, K., Stangl, G., Milev, I. (2010). Investigation of Geodynamics of Central and Eastern Europe, Balkan Peninsula and Bulgaria. In: Monograph "Intelligent Data Analysis in Global Monitoring for Environment and Security", ITHEA, Kiev-Sofia, pp. 92–108.

Mitas, L., Mitasova, H. (2005). Spatial Interpolation. Chapter 34 in: P. A. Longley, M. F. Goodchild, D. J. Maguire, D.W. Rhind (eds) Geographical Information Systems: Principles, Techniques, Management and Applications. 2nd Edition. New Jersey: John Wiley & Sons.

NIMA TR 8350.2. (1997). Technical Reports. Third Edition. Department of Defence W.G.S-84.

OGP (International Association of Oil & Gas Producers). (2006). Surveying and Positioning Guidance Note, number 7, part 2. Coordinate Conversions and Transformations Including Formulas.

Reilinger, R.E., Ergintav, S., Bürgmann, R., McClusky, S., Lenk, O., Barka, A., Gurkan, O., Hearn, L., Feigl, K. L., Cakmak, R., Aktug, B., Ozener, H., Töksoz, M.N. (2000). Coseismic and postseismic fault slip for the 17 August 1999, M=7.5, Izmit, Turkey Earthquake, Science, 289, 1519-1524.

Ruffhead, A. C. (2017). Introduction to multiple regression equations in datum transformations and their reversibility. Survey Review, in press. DOI: http://dx.doi.org/10.1080/00396265.2016.1244143

Schaback, R. (2007). A Practical Guide to Radial Basis Functions, April 16, 2007. http://num.math.uni-goettingen.de/schaback/teaching/sc.pdf

Soycan, M. (2005). Polynomial versus similarity transformations between GPS and Turkish reference systems. Survey Review, 38 (295), 58–69. DOI: http://dx.doi.org/10.1179/sre.2005.38.295.58

Soycan, M., Soycan, A. (2008). Transformation 3D GPS Cartesian coordinates to ED50 using polynomial fitting by robust re-weighting technique. Survey Review, 40 (308), 142–155. DOI: http://dx.doi.org/10.1179/003962608X253673

Soycan, M., Soycan, A. (2014). Comparison of Several Techniques for Fitting of the EGM08 to GPS/Leveling Datum. Arabian Journal for Science and Engineering, 39 (7), 5637–5651. DOI: http://dx.doi.org/10.1007/s13369-014-1136-1

Talmi, A., Gilat, G. (1977). Method for smooth approximation of data. Journal of Computational Physics, 23 (2), 93–123. DOI: http://dx.doi.org/10.1016/0021-9991(77)90115-2

Tokhey, M. E. (2000). On the Determination of Consistent Transformation Parameters between GPS and the Eqyptian Geodetic Reference Systems. International Symposium on Gravity, Geoid and Geodynamics.

TUTGA-99A (Turkey National Fundamental GPS Network-1999A) (2002). General Command of Mapping. Harita Dergisi, Special Issue.

Vaníček, P., Steeves, R. R. (1996). Transformation of coordinates between two horizontal geodetic datums. Journal of Geodesy, 70 (11), 740–745. DOI: http://dx.doi.org/10.1007/BF00867152

Wahba, G. (1990). Spline models for observational data. CNMSNSF Regional conference series in applied mathematics 59. Philadelphia: SIAM.

Zeng, H. (2014). Planar coordinate transformation and its parameter estimation in the complex number field. Acta Geodaetica et Geophysica, 49 (1), 79–94. DOI: http://dx.doi.org/10.1007/s40328-014-0040-1

URL-1: http://itrf.ensg.ign.fr/general.php

URL-2: http://inspire-regadmin.jrc.ec.europa.eu/dataspecification/themes/gg/Chapter5.pdf

URL-3: http://www.crs-geo.eu/nn_124226/crseu/EN/CRS__Description/crs-national__node.html?__nnn=true

Soycan M., Soycan A., Tunalıoğlu N. (2017). Transformacija deformiranih geodetskih mrež v nove referenčne koordinatne sisteme: študija primera transformacije ED50-ITRFXX v Turčij. Geodetski vestnik, 61 (1): 58-75. DOI: 10.15292/geodetski-vestnik.2017.01.58-75

prof. Metin Soycan, Ph.D.
Yildiz Technical University, Faculty of Civil Engineering
Department of Geomatic Engineering
Davutpasa Campus TR-34220 Esenler-İstanbul-Turkey
e-mail: soycan@yildiz.edu.tr

assoc. prof. Arzu Soycan, Ph.D.
Yildiz Technical University, Faculty of Civil Engineering
Department of Geomatic Engineering
Davutpasa Campus TR-34220 Esenler-İstanbul-Turkey
e-mail: topbas@yildiz.edu.tr

assoc. prof. Nursu Tunalıoğlu, Ph.D.
Yildiz Technical University, Faculty of Civil Engineering
Department of Geomatic Engineering
Davutpasa Campus TR-34220 Esenler-İstanbul-Turkey
e-mail: ntunali@yildiz.edu.tr