

KARTE VREDNOSTI STAVBNIH ZEMLJIŠČ – METODOLOŠKI PRISTOP

URBAN LAND VALUE MAPS – A METHODOLOGICAL APPROACH

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IZVLEČEK

Karte vrednosti zemljišč se običajno uporabljajo pri množičnem vrednotenju nepremičnin za določitev davka na nepremičnine. Zaradi kompleksnosti procesov upravljanja nepremičnin pa se grafični prikazi vrednosti zemljišč lahko uporabljajo tudi za različne druge namene, ki niso pogojeni z zakonskimi zahtevami. V tej študiji je predlagan koncept razvoja karte vrednosti zemljišča, ki se lahko uporablja za nedavčne namene. Predlagana karta je bila razvita s statističnimi in geostatističnimi metodami. Razvit je bil referenčni sloj, ki ustreza reprezentativnemu zemljišču, uporabljeni so bili statistični modeli za določitev faktorjev, s katerimi se prilagajajo vrednosti zemljišča glede njegovih prostorskih značilnosti. Predstavljen je teoretični koncept, ki je uporabljen za razvoj karte vrednosti zemljišč za mesto Olsztyn na severovzhodu Poljske.

ABSTRACT

Land value maps are generally used in mass appraisal for the determination of property taxes. In view of the complex nature of property management processes, land value maps may also serve a variety of purposes which are not dictated by legal requirements. This study proposes a concept for the development of a land value map which may be applied for non-tax purposes. The proposed map was developed with the use of statistical and geostatistical methods. A reference layer corresponding to a representative property was developed, and statistical models were used to determine coefficients that adjust property value in view of its non-spatial attributes. A theoretical concept was presented, and it was used to develop a land value map for the city of Olsztyn in north-eastern Poland.

KLJUČNE BESEDE

karta vrednosti, geostatistika, kriging, prostorski modeli, množično vrednotenje, Poljska

KEY WORDS

value map, geostatistics, kriging, spatial models, mass appraisal, Poland

1 INTRODUCTION

Information about property prices and values is one of the key factors that determine effective decision-making in various types of organized activity, in particular activities that rely on spatial data. Such information can be stored and released in the form of land value maps which are generally used in mass appraisal for the determination of property taxes (Kryvobokov, 2004; Chapman, 2009). The process of developing maps for tax purposes has to conform to national laws and regulations, and it involves the acquisition and processing of large amounts of spatial and legal data, which is costly and time-consuming. Maps which are developed not only for tax purposes, but which have a variety of other legal, investment and practical uses, can be a source of information about land value. The rapid development of geoportals triggers the development of such cartographic materials (Maguire and Longley 2005). Maps are generally developed based on the results of detailed market analyses of individual cities (Colwell and Munneke 1997, 2003; Guntermann and Thomas 2005; Haughwout et al., 2008; Bryan and Sarte, 2009). The property market is shaped by complex spatial and non-spatial processes which exert a combined effect on market value (Munroe, 2007). Prices and values are determined not only by the location of property, but also by a variety of exogenous and endogenous factors which have to be taken into account in market analysis (Isakson, 1997; Galster et al., 2004; Earnhart, 2006; Palmquist, 2005; Zrobek and Grzesik, 2013). Location factors which contribute to the cartographic representation of property value are identified in the analytical process.

The concept of maps for tax purposes dates back to ancient times, whereas the first land value maps were developed in the early 20th century (Batt, 2009). David Ricardo and Johann Heinrich von Thunen were the first economists to observe that land value is determined by the spatial structure of cities and can be represented by mathematical models that can be verified empirically in the process of market development. Such models have been proposed by Colwell and Munneke (1997, 2003), Palmquist (2005) as well as Bryan and Sarte (2009) who argued that land value maps are a valuable source of information that can serve a variety of purposes unrelated to taxation. Land value maps do not have to comply with strict formal and legal requirements if they are used not only for the determination of property taxes. Modern land value maps rely on Geographic Information System (GIS) systems which offer sophisticated tools for spatial data analysis (Anselin, 1998; Clapp and Rodriguez, 1998; McCluskey et al., 2000; Burrough, 2001; Vickers and Thurstain-Goodwin, 2002; Gall, 2006; Cichocinski, 2008; Cotteleer et al., 2008). Most maps contain information about the prices of urban (Haughwout et al., 2008) and agricultural land (Hite et al., 1999). According to Batt (2009), the majority of land value maps have been developed for the United States, whereas fewer cartographic resources are available for Europe, Asia and Africa. Attempts are also made to create land value maps for cities with weakly developed property markets (Weiss, 2005; Ping, 2005; Waljiyanto and Suryohadi, 2004; Aleksiene and Bagdonavicius, 2005).

Maps presenting land prices and values are created with the use of various methods and tools. Howes (1980) discusses numerous examples of land value maps prepared for the needs of one-off research or development projects which cover only small fragments of cities. In line with the existing methodology, the majority of land value maps rely on the correlations between land value and distance from central business districts (CBD). Liu, Zheng, Huang and Tang (2007) analyzed the evolution of land prices and values based on the distance separating the property from CBD, public facilities and schools. In the

approach proposed by Bugs (2007), land value maps were developed based on zones reflecting the distance between the evaluated property and the city centre, principal avenues, health centres and high-risk flood areas. Spatial analyses and GIS tools are deployed to generate maps illustrating land use patterns and land value in urban environments.

Hedonic models where selected property attributes are the main price determinants play an important role in a different field of research focusing on cartographic visualization of land value (Kelley et al., 1998; Benjamin et al., 2004; Noelwah, 2005; Cotteleer et al., 2008; Hannoen, 2008; Páez, 2009; Montero and Larraz, 2010). Theory and practice indicate that models which do not rely on spatial autocorrelations may produce distorted results (Anselin, 1998; Basu and Thibodeau, 1998; Tu et al., 2007), and for this reason, many authors recommend the use of spatial models in market analyses and price predictions (Can and Megbolugbe, 1997; Bowen et al., 2001; Valente et al., 2005; Bourassa et al., 2007, 2010; Walacik et al., 2013). GIS tools and geostatistical methods are increasingly often deployed to model a price surface for land. Fik et al. (2003) combined hedonic models and surface trends in LVS (Location Value Signature) analyses. Bourassa et al. (2010) also recommended the combined use of hedonic models and geostatistical methods. Geostatistical methods can supplement traditional statistical analyses to account for the spatial distribution of the examined phenomena. Geostatistical methods are far less popular in property market analyses than other statistical approaches. Their application may be fraught with certain difficulties, such as the need to fulfil fundamental requirements concerning the size of the dataset, data distribution and, above all, the stationary character of data (both first-order and second-order stationarity). In this paper, hedonic models were combined with geostatistical methods to develop an urban land value map of the city of Olsztyn in north-eastern Poland.

2 METHODOLOGICAL CONCEPT

The property market is highly complex, and traded properties differ in location as well as non-spatial attributes (area, shape, infrastructure). The influence of the evaluated attributes will be assessed with the use of statistical models, and geostatistical methods will be deployed to analyze the spatial distribution of land values. The development of mass appraisal algorithms may also require the determination of uniform zones – areas in which properties characterized by the same attribute scores have identical value. The proposed methodological concept can be divided into the following stages:

1. collection of input data,
2. development of regression models illustrating the correlations between prices and selected property attributes,
3. estimation of the reference value (reference layer) for a representative property,
4. estimation of adjusting coefficients that account for similarities with the representative property,
5. division of the analyzed area into uniform zones and determination of land values in each zone.

The reference layer illustrates the spatial distribution of values of property with strictly defined non-spatial attributes, and it plays a very important role in the proposed method. The reference layer will apply to the representative property, i.e. property where explanatory variables take on zero value, which requires the development of a corresponding measurement scale. The representative property is privately-owned plot of vacant land, zoned for housing construction and equipped with basic infrastructure

(water, electricity and gas supply, sewage collection), with the estimated area of 800 m². The estimated value is influenced by non-spatial attributes as well as the location of the analyzed property, which can be expressed, according to own considerations, as:

$$W_{ref} = W_B + W_L, \quad (1)$$

where:

W_B - value of representative property when location is disregarded (base value),

W_L - effect of location.

The spatial dependencies between location and property prices can be evaluated based on the results of spatial interpolation by ordinary kriging. According to own considerations and general formula of kriging (Isaaks and Srivastava, 1998), if for every location s_0 :

$$W_B(s_0) = \hat{Y}(s_0), \text{ and } W_L(s_0) = \sum_{i=1}^n w_i(s_0) \varepsilon(s_i), \quad (2)$$

where:

$W_B(s_0)$ - base value at point s_0 ,

$\hat{Y}(s_0)$ - theoretical value of representative property resulting from the regression model at point s_0 ,

$W_L(s_0)$ - effect of location at point s_0 ,

w_i - kriging weights,

$\varepsilon(s_i)$ - model residual in location I ,

the two principal data analysis components can be integrated into a single model which accounts for the correlations between the results of an econometric model and kriging estimation methods. The regression-kriging model for estimating the reference value, according to own considerations, takes on the following generalized form:

$$W_{ref}(s_0) = \hat{Y}(s_0) + \sum_{i=1}^n w_i(s_0) \varepsilon(s_i), \quad (3)$$

where $W_{ref}(s_0)$ is the reference value (value of representative property) at point s_0 .

In the regression-kriging model, various methods for determining theoretical value can be considered by relying on the following regression models:

- multiple regression model (linear and non-linear),
- spatial autoregressive (SAR) model,
- geographically weighted regression (GWR) model.

When a multiple regression model is used, spatial correlations can be illustrated by a variance-covariance matrix developed based on a semivariogram of the residuals, therefore, parameters are estimated in successive iterations with the use of the generalized least squares method. In the SAR model, the above correlations are presented by a spatial structure matrix and a spatial autocorrelation index. The GWR

model accounts for the discussed correlations with the use of weights determined based on the kernel density function. The reference layer of a directional land value map was developed with the use of the models described in Table 1.

Table 1: Models applied in the process of developing the reference layer of a directional land value map.

No.	Model	Method for analyzing spatial correlations	Symbol
1	Additive linear multiple regression model	None	OLS
2	Multiplicative exponential multiple regression model	None	OLSN
3	Additive linear multiple regression model	Covariance matrix based on a semivariogram of the residuals	GLS
4	Multiplicative exponential multiple regression model	Covariance matrix based on a semivariogram of the residuals	GLSN
5	Spatial autoregressive model	Autocorrelation of the residuals based on the spatial structure matrix with inverse distance weighting	SAR
6	Additive linear geographically weighted regression model	Weights determined based on the kernel density function	GWR

Each of the listed models feature a constant which can be interpreted as the theoretical value of the explained variable on the assumption that explanatory variables take on zero value. If we assume that the modelled property is characterized by such attribute values, the result according to own considerations is:

$$W_{ref}(s_0) = \beta_0 + \sum_{i=1}^n w_i(s_0) \varepsilon(s_i), \tag{4}$$

where β_0 is a constant explained as theoretical value of property characterized by attributes the same as modelled property. Spatial autoregressive models have to additionally account for the autocorrelation of the explained variable or the autocorrelation of the residuals. The type of SAR model (spatial lag or spatial error model) is selected based on the results of the Lagrange Multiplier (LM) test. The GWR approach produces a series of models independently for every point which, in this case, corresponds to the geometric centre of the traded land plot. Therefore, constant β_0 in the value estimation formula will refer independently to each centroid of the analyzed land plots.

In this study, non-linear models (OLSN and GLSN) take on the multiplicative exponential form. In this case, the semivariogram for building the covariance matrix in the GLSN model will not apply to the residuals but to the log of error component ξ . In the group of factors that determine the value of property, some have a non-spatial (e.g. geometric configuration of a land plot) or spatial character if they are indirectly or directly related to location. In the case of variable characterized by continuity in space the analyzed attribute (or attributes) is disregarded in a model, and cokriging methods are then applied where the omitted attribute is used as an additional variable. The regression-cokriging model for estimating the reference value can take on the following form on the assumption that one main variable (model residual) and one additional variable are taken into account, what is extension of formula (3) with formula of cokriging (Isaaks and Srivastava, 1998):

$$W_{ref}(s_0) = \hat{Y}(s_0) + \sum_{i=1}^n w_i(s_0) \varepsilon(s_i) + \sum_{j=1}^m w_j(s_0) Z_d(s_j), \quad (5)$$

or, when multiplicative models are used, according to own research:

$$W_{ref}(s_0) = \hat{Y}(s_0) \cdot \left(\sum_{j=1}^n w_j(s_0) \xi(s_j) + \sum_{i=1}^m w_i(s_0) Z_d(s_i) \right), \quad (6)$$

where $Z_d(s_j)$ is the value of an additional variable at point s_j . The discussed model can be generalized (at least theoretically) for use with any number of additional variables, but in practice, up to three variables are used (a high number of cross-semivariograms have to be estimated). The theoretical value is estimated by determining constant β_0 in OLS, OLSN, GLS and GLSN models, and, additionally, the *signal* component in SAR models. In the GWR model, the constant will be determined independently for every model at points where parameters were estimated.

In additive models, according to the authors, land value can be expressed by the following formula:

$$W_k = W_{ref} + \sum_{i=1}^m k_i X_i, \quad (7)$$

where coefficients k_i represent the absolute (quota) adjustment per unit of change in the value of attribute X_i .

In multiplicative models, according to the authors, the following formula is applied to determine land value:

$$W_k = W_{ref} \cdot \prod_{i=1}^m (1 + r_i X_i), \quad (8)$$

where coefficients r_i represent the relative adjustment per unit of change in the value of attribute X_i .

In the analyzed case, a simplified approach was used, and compound percentages were replaced with standard percentages. This is a justified solution if the values of property attributes do not vary considerably. Adjusting coefficients were determined by the least squares method and the quasi-Newton algorithm which approximates second-order derivatives of the loss function (difference between actual and theoretical value) to calculate the minimum value (Fletcher 1987). The reference value can be determined from the reference layer based on raster values at points relating to the analyzed transaction (centroids of land plots).

Land values in the city were presented in a cartogram where the analyzed area was divided into uniform zones. The average value of land in every zone corresponds to the average value of the reference layer adjusted for the attributes of representative property in the analyzed zone. A uniform zone is a continuous area which is enclosed by legal or administrative boundaries and characterized by similar land values. Uniform zones were identified based on several criteria, the most important being:

- identical (or similar) land value per unit of area,
- identical (or similar) function in the local land use plan,
- identical functional and spatial features,
- land value is influenced by similar local factors.

Regardless of the adopted approach, special zones may have to be established due to special ecological, geotechnical and geological requirements, including for land plots occupied by roads and water courses.

3 DATA COLLECTION AND EVALUATION

Property transactions involving vacant land plots in Olsztyn, a city of about 200 000 inhabitants in north-eastern Poland, were analyzed. The spatial arrangement of the city of Olsztyn is complex, which is a result of the natural conditions (numerous lakes and forest areas), as well as of historical factors. The central part of the city is surrounded by residential districts of blocks of flats, typical of the 1960s and 1970s, as well as residential districts of detached houses. The urban area is divided into sectors, which were created in the process of planning. The evaluated set of data concerned approximately 400 transactions concluded in 2009-2011. Data was obtained from the Register of Property Prices and Values kept by the Olsztyn City Office. The following property variables were analyzed:

- land use,
- ownership status (ownership and perpetual usufruct),
- infrastructure,
- geometric configuration of land plot (area and shape),
- maximum noise levels.

Locations of Olsztyn and locations of transactions are shown in Figure 1.



Figure 1: Olsztyn on the background of Poland and locations of transactions in the area of the city.

Only high-density or low-density residential properties (total of 293 transactions) were used for developing the reference layer. Information about non-residential property was used only for calculating the adjusting coefficients to base value. Transaction data, including property prices and attributes, was verified before analysis. Preliminary analyses revealed that the hypothesis postulating an absence of correlations between selected variables and transaction prices (at significance level of 0.05) cannot be rejected. The above was reported for maximum noise levels and infrastructure. Noise

levels constitute one of many property-specific attributes and a spatial factor which is taken into account in interpolation, which is why this variable was disregarded in further analyses. No significant correlations between noise levels and property prices were reported in an analysis of variance. With regard to infrastructure, only one variable describing this attribute was taken into account. Its value was expressed by the arithmetic mean of values assigned to specific utility services. The final model comprised four explanatory variables: land use X_1 (high-density or low-density housing), ownership status X_2 (ownership or perpetual usufruct), infrastructure – X_3 and geometric configuration of land plot – X_4 . In the adopted measurement scale, zero values corresponded to representative property, i.e. land zoned for low-density housing, privately owned, with full infrastructure access, attractive plot shape and area.

A digital map containing infrastructure data and a noise map of Olsztyn were used in the analysis. Calculations and statistical analyses were performed in R, Statistica v. 10 and ArcInfo v. 10.0. applications.

4 RESULTS

A total of 6 regression models listed in Table 1 were estimated in the analysis. Different parameter estimation methods had to be used to account for the specific requirements of each model. The least squares method is generally applied in traditional regression models (OLS, OLSN, GWR), whereas models that account for mutual spatial correlation of explanatory variables (GLS and GLSN) rely on the generalized least squares technique. Parameters are estimated by iteration (Schabenberger and Gotway, 2005), and according to Kitanidis (1994), even a single iteration can produce satisfactory results. This study involved three iterations after which the estimated parameters remained practically unchanged. The use of the least squares method in SAR models produces biased and inconsistent estimators (Anselin, 1999), which is why the maximum likelihood method was used. The parameters estimated in each model are presented in Tables 2 through 7.

Table 2. Parameters estimated in the OLS model.

Analytical expression: $Y = \beta_0 + \sum X\beta + \varepsilon$				
Ordinary least squares method (OLS)				
	Coefficient	SEE	t	p-value
Constant	295.026	8.029	36.746	0.000
X_1	170.230	21.498	7.918	0.000
X_2	-68.848	31.968	-2.154	0.032
X_3	-137.007	12.319	-11.121	0.000
X_4	-20.846	13.318	-1.565	0.119

F = 52.29 (4, 289), $p < 0.001$, Standard error of the estimate: 81.770

$R^2 = 0.420$, adjusted $R^2 = 0.412$, AIC = 3395.40

Table 3: Parameters estimated in the OLSN model.

Analytical expression: $y = \beta_0 \cdot \prod \beta^x \cdot \xi$
 Ordinary least squares method (OLS)

	Coefficient ($e^{\ln\beta}$)	SEE	t	p-value
Constant	227.221	1.387	199.876	0.000
X_1	1.687	0.243	6.937	0.000
X_2	0.790	0.376	2.103	0.036
X_3	0.576	0.045	12.779	0.000
X_4	0.946	0.795	1.188	0.236

F = 59.15 (4, 289), p < 0.001, Standard error of the estimate: 81.531
 R² = 0.450, adjusted R² = 0.442, AIC = 127.55

Table 4: Parameters estimated in the GLS model.

Analytical expression: $Y = \beta_0 + \sum X\beta + \varepsilon$
 Generalized least squares method (GLS), restricted maximum likelihood method (REML)
 Spatial correlations were estimated based on a semivariogram of the residuals.
 Number of iterations: 3

	Coefficient	SEE	t	p-value
Constant	305.564	27.793	10.994	0.000
X_1	159.721	27.185	5.875	0.000
X_2	-36.892	39.888	-0.925	0.356
X_3	-130.894	26.154	-5.005	0.000
X_4	-37.061	12.435	-2.980	0.003

Spherical model: nug = 1071.12, sill = 7455.54, range = 2156.25
 Standard error of the estimate: 128.03
 AIC = 3264.48, logLik = -1626.24

Table 5: Parameters estimated in the GLSN model.

Analytical expression: $y = \beta_0 \cdot \prod \beta^x \cdot \xi$
 Generalized least squares method (GLS), restricted maximum likelihood method (REML)
 Spatial correlations were estimated based on a semivariogram of the residuals.
 Number of iterations: 3

	Coefficient ($e^{\ln\beta}$)	SEE	t	p-value
Constant	275.619	4.858	56.736	0.000
X_1	1.658	0.318	5.217	0.000
X_2	0.875	0.931	0.940	0.348
X_3	0.607	0.113	5.362	0.000
X_4	0.898	0.370	2.428	0.016

Spherical model: nug = 1071.12, sill = 7455.54, range = 2156.25
 Standard error of the estimate: 82.215
 AIC = 6.270, logLik = 2.86

Table 6: Parameters estimated in the SAR model.

Analytical expression (spatial error model): $y = X\beta + \lambda W\varepsilon + \xi$, (based on the results of the LM test)
 Adjacency matrix based on inverse distance weights
 Maximum likelihood method

	Coefficient	SEE	z	Pr (> z)
Constant	312.844	34.911	8.961	0.000
X ₁	140.891	21.383	6.589	0.000
X ₂	-61.947	31.004	-1.998	0.045
X ₃	-161.933	18.758	-8.633	0.000
X ₄	-28.813	13.178	-2.186	0.029

LM test:

LMerr = 90.76, p < 0.001, LMlag = 56.80, p < 0.001

RLMerr = 35.57, p < 0.001, RLMLag = 1.61, p = 0.204

$\lambda = 0.879$

Standard error of the estimate: 71.677

AIC = 3375.70, logLik = -1680.851

Table 7: Parameters estimated in the GWR model.

Analytical expression: $Y = \beta_0 + \Sigma X\beta + \varepsilon$
 Geographically weighted regression method (GWR)

	Average	Min	Max	SD
Constant	309.342	240.582	359.542	35.966
X ₁	137.840	10.552	208.672	70.606
X ₂	-84.300	-160.589	59.218	47.268
X ₃	-158.865	-256.104	-123.060	30.463
X ₄	-36.900	-59.685	18.599	24.267

Bandwidth = 2446.30

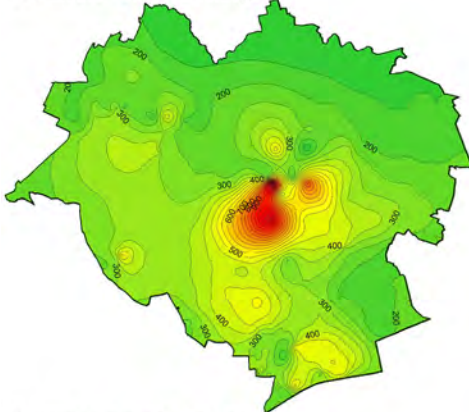
Standard error of the estimate: 69.545

R² = 0.574, adjusted R² = 0.539, AIC = 3370.81

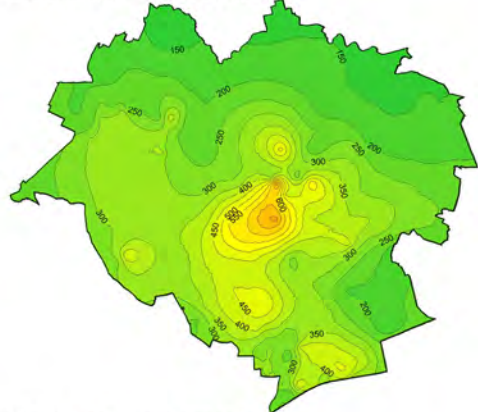
The above models were developed primarily for the purpose of estimating the reference value of land. The constant values and their verification and statistical analysis play the most important role. The constant was the most statistically significant parameter in all the models. When various estimation methods are used, the coefficient of determination cannot be applied as a criterion for model evaluation. The Akaike Information Criterion (AIC) can be used, and it demonstrates that non-linear models have a clearly better fit than linear models.

Modelled results should be interpreted in view of the fact that the location effect, which was not taken into account in the first two models (OLS and OLSN) and was considered only indirectly in the remaining models (in parameter estimation), was the main cause of significant variation in the residuals. The quality of the fit is also indicative of the degree of “flattening” of transaction prices and land values. The greater the standard error of the estimate, which is measured by the standard deviation of the residuals,

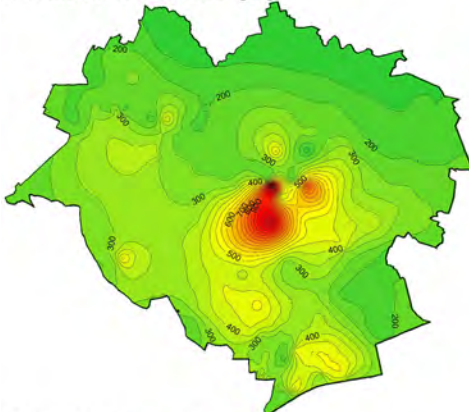
OLS model + residual cokriging



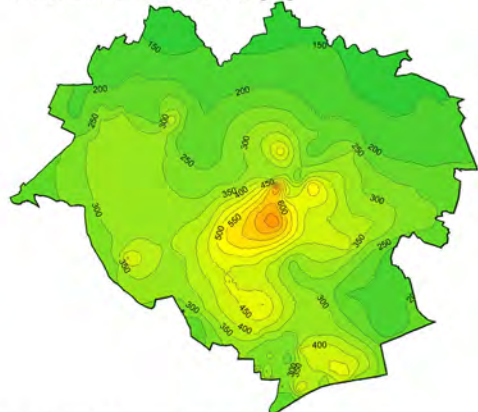
OLSN model + residual cokriging



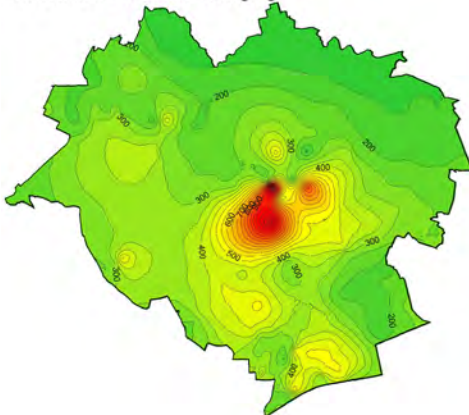
GLS model + residual cokriging



GLSN model + residual cokriging



SAR model + residual cokriging



GWR model + residual cokriging

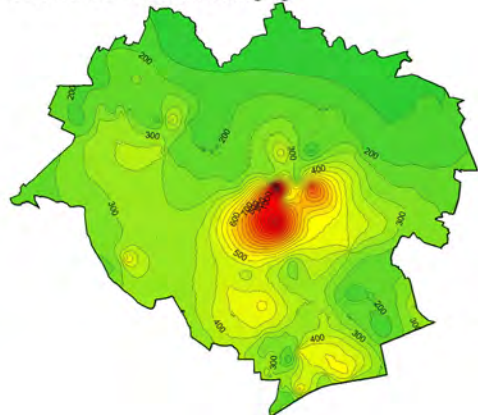


Figure 2: Spatial distribution of reference values based on the adopted models and interpolation of the residuals by cokriging methods (value in PLN, where 1 PLN is about EUR 0.24).

the greater the variation in prices due to, among others, the location effect. In this case, price variability is explained only by the variation of explanatory variables which have a non-spatial character.

Development density in the neighbourhood was used as an additional variable for interpolation by cokriging methods. This attribute is not directly related to the appraised property, but it concerns the neighbourhood (the properties analyzed in this study comprised vacant land, therefore development density was always zero) and constitutes one of many location factors. The decision not to use this variable directly in the model was also prompted by the fact that individual assessments are more likely to rely on information about maximum development density prescribed by the local land use plan. The adopted method of determining development density (kernel density estimation) is continuous and characterized by spatial autocorrelation. The reference values determined with the use of different spatial models (based on the formulas given in chapter 2) are presented in Figure 2. Ordinary cokriging methods and spherical semivariograms were used for interpolation.

The results obtained reveal highly similar distributions of reference values in linear and non-linear models. In central districts characterized by high property values, base values are somewhat lower in non-linear than in linear models. The differences between models can be attributed to the fact that in linear models, adjustments are absolute and are not directly determined by “base value”. The latter leads to the risk of “base value” overestimation in high-price locations or underestimation in locations characterized by negative residuals with high absolute value. The adjustments made in linear models are relative, and therefore, the quota effect of explanatory variables will be dependent on “base value”. Several factors have to be taken into consideration when selecting the most appropriate model for developing the reference layer of a land value map. Above all, the model should fulfil the basic functional and statistical requirements, which is not always achievable. Linear models are recommended where the analyzed properties are highly similar, in particular with regard to price. The standard error of the estimate (measured by the standard deviation of the residuals) should be minimal. A model can also be selected by evaluating the coefficient of determination R^2 (when the least squares method is applied), the maximum likelihood estimator (when the maximum likelihood method is applied) or AIC (when various estimation methods are used). The final decision should be made after comparing directional values with transaction prices. The adjusting coefficients for reference values estimated with the use of formulas (7) and (8) are presented in Table 8.

Table 8: Adjusting coefficients for reference values.

	W_{OLS} k_i [PLN/m ²]	W_{OLSN} r_i [%]	W_{GLS} k_i [PLN /m ²]	W_{GLSN} r_i [%]	W_{SAR} k_i [PLN /m ²]	W_{GWR} k_i [PLN /m ²]
Land use	138.87	48.55	128.23	47.58	110.62	117.08
Ownership	-71.30	-22.55	-44.80	-17.33	-67.43	-94.73
Infrastructure	-134.78	-41.30	-129.75	-38.49	-159.45	-160.20
Configuration	-24.90	-9.64	-37.11	-12.12	-31.61	-31.49
R^2	0.912	0.811	0.910	0.821	0.907	0.819
SEE	31.60	46.29	32.01	45.02	32.58	45.39

Where 1 PLN is about EUR 0.24

All the coefficients were significant at p-value less than 0.001. In the group of linear models, the multiple regression model, where the reference value was estimated by the ordinary least squares method, produced the best fit. In the group of non-linear models, a better fit was produced when the reference value was estimated with the use of the GLSN model rather than the OLSN model.

The concept of “estimation precision” in the valuation process may raise controversy because price is not a physical attribute that can be measured with the risk of error. For this reason, even advanced error analysis methods can produce unreliable results. In this case, the degree to which the estimated directional value fits the market data can be estimated with the simplest metrics (e.g. standard deviation of differences between directional value and transaction price). In the analyzed case, the average standard deviation of the residuals was determined in the range of PLN 31.60/m² (EUR 7.58/m²) to PLN 46.29/m² (EUR 11.11/m²), i.e. between 13.1% and 19.2% of the average price.

The degree of fit between estimated directional values and transaction prices is determined by various factors, including:

- availability of transaction data,
- reliability of transaction prices,
- number of variables and choice of scale for measuring property attributes,
- dispersion of property transactions,
- spatial autocorrelation in prices,
- correlation between prices and an additional variable for cokriging,
- semivariogram model for kriging (cokriging).

The calculation of adjusting coefficients can be problematic for land that has been zoned not only for residential construction but also for other purposes. When additional land functions are directly included in regression models as variables, the results of the analysis can be significantly warped because individual attributes are characterized by various weights in different functions (for example, infrastructure is not a highly important consideration in green areas). For this reason, adjusting coefficients were estimated based on reference layers. It was assumed that the correlations between the prices of property with different zoning functions have a relative (and not a quota) character. Layers W_{OLSN} and W_{GLSN} were used in calculations. Coefficients r_i were estimated by determining the correlations between the prices of property with the same function and the reference value of low-density residential property by using a simplified formula:

$$W_k = W_{ref} \cdot (1 + r_i), \tag{9}$$

where the adjusting coefficient is determined based on the unit price rather than the directional value of property. The resulting coefficients for reference layers W_{OLSN} and W_{GLSN} are presented in Table 9.

Table 9: Adjusting coefficients for different land zoning functions (in relation to low-density housing)

Function	W_{OLSN} r_i [%]	W_{GLSN} r_i [%]
Commerce	88.79	88.67
Industry	-31.99	-32.85
Transport	-22.64	-22.29
Agriculture	-86.58	-86.58
Greens	-70.13	-70.51

Source: own study.

The results obtained for two reference values are highly similar. The coefficients are very easy to interpret. For example, the coefficient for industrial property indicates that the average price of industrial land is approximately 32% lower in comparison with the reference value (i.e. the value of land zoned for low-density residential construction). The coefficients shown in Tables 8 and 9 next to the reference value can be used to calculate average values for land value maps.

The analyzed area was divided into zones of similar value based on the functional and spatial structure of the city, zoning designation, type of land use and property prices. In this study, the corresponding zones were determined based on the following assumptions:

1. The following types of land were excluded from the study because their value could not be directly estimated based on the available transaction data:
 - land occupied by water bodies (the largest lakes within the city's administrative boundaries),
 - dense forests,
 - idle land.
2. Zone boundaries should be determined in view of the city's functional and spatial layout in the zoning plan.
3. Zone boundaries should run parallel to major roads in the city (Matthews and Turnbull 2007).
4. Zones should consist of land with identical or similar zoning designation.
5. Zone boundaries should account for the variation in prices (in this case, the reference value of land).

The analyzed area was divided into 109 zones based on the discussed criteria. In each zone, basic reference value statistics were determined from the raster layer with the use of coefficient W_{OLSN} . The average value for each zone is an estimation of the reference value. It can also be determined by block kriging (or cokriging) which is an equivalent method of determining the average value of a variable within a large area. The average values estimated for each zone in view of the predominant land use functions were used to determine the final average value of land.

5 CONCLUSIONS

In this study, statistical and geostatistical methods were used to develop a map of average land values. The combined use of the above methods for variations inside the dataset and spatial distribution of data. The proposed method of developing land value maps based on the reference layer of representative property facilitates the estimation of average land values at any point in the analyzed area. A land value map is a valuable source of information for a variety of market surveys which investigate the social and economic aspects of property management and administration. Such surveys are carried out by territorial governments as well as central administration agencies to generate many types of statistical data. Two types of objectives are generally pursued: short-term goals which are related to market restructuring efforts, and long-term goals which involve regular monitoring of prices as an effective tool for managing land resources. Both goals require instant access to objective information about property prices and values which affect supply prices on the local or regional property market. A land value map is a useful tool for controlling economic processes at different levels of administration. Short-term goals include:

- determining the value of property owned by the State Treasury and territorial governments,
- determining the effect of land value on property prices,

- acquiring land for urban planning decisions,
- determining land prices for investors in view of the characteristic features of the local market,
- developing preferential land policies for investors,
- determining the property market's growth prospects and using that information for taxation purposes;

Long-term goals include:

- economically justified planning decisions,
- determining planning and development conditions based on the revenues generated from the lease of land owned by the State Treasury and territorial governments,
- creating favourable conditions for development and investment,
- monitoring the prices of traded property and developing property assessment maps for taxation purposes,
- providing the public with fast and unrestricted access to information about land prices and values via the Internet.

Land value maps developed based on the proposed approach serve a variety of purposes and constitute a priority resource for a wide spectrum of activities on the property market. Suggestion for further research can be explaining the character of variability of land value (continuous or discontinuous) and on this basis, the division of areas into zones of similar values. Such maps could be more informative for the recipient.

References:

- Aleksiene, A., A. Bagdonavicius, A. (2005). GIS and Land Value Mapping. The Lithuanian Experience. *International Association of Assessing Officers: Fair & Equitable*.
- Anselin, L. (1998). GIS Research Infrastructure for Spatial Analysis of Real Estate Markets. *Journal of Housing Research*, 9 (1), 113–133.
- Anselin, L. (1999). *Spatial Econometrics*. Bruton Center School of Social Sciences, University of Texas at Dallas.
- Basu, S., Thibodeau, T.G. (1998). Analysis of Spatial Autocorrelation in House Prices. *The Journal of Real Estate Finance and Economics*, 17 (1), 61–85. DOI: <http://dx.doi.org/10.1023/A:1007703229507>
- Batt, H. W. (2009). Land Value Maps are Not New, But Their Utility Needs to be Re-Discovered. *International Journal of Transdisciplinary Research*, 4 (1), 108–158.
- Benjamin, J. D., Randall, S., Guttery, R. S., Sirmans, C.F. (2004). Mass Appraisal: An Introduction to Multiple Regression Analysis for Real Estate Valuation. *Journal of Real Estate Practice and Education*, 7 (1), 65–77.
- Bourassa, S. C., Cantoni, E., Hoesli, M. (2007). Spatial Dependence, Housing Submarkets, and House Price Prediction. *The Journal of Real Estate Finance and Economics* 35 (2), 143–160. DOI: <http://dx.doi.org/10.1007/s11146-007-9036-8>
- Bourassa, S. C., Cantoni, E., Hoesli, M. (2010). Predicting House Prices with Spatial Dependence: A Comparison of Alternative Methods. *Journal of Real Estate Research*, 32 (2): 139–159.
- Bowen, W. M., Mikelbank, B. A., Prestegaard, D. M. (2001). Theoretical and Empirical Considerations Regarding Space in Hedonic Housing Price Model Applications. *Growth and Change*, 32 (4): 466–490. DOI: <http://dx.doi.org/10.1111/0017-4815.00171>
- Bryan, K. A., Sarte, P. G., (2009). Semiparametric Estimation of Land Price Gradients Using Large Data Sets. *Federal Reserve Bank of Richmond Economic Quarterly*, 95 (1): 53–74.
- Bugs, G. (2007). Urban land value map: a case study in Eldorado do Sul – Brazil. GIS applications, Instituto Superior de Estatística e Gestão da Informação, Universidade Nova de Lisboa – UNL, Master of Science on Geospatial Technologies.
- Burrough, P. A. (2001). GIS and geostatistics. Essential partners for spatial analysis. *Environmental and Ecological Statistics*, 8, 361–377. DOI: <http://dx.doi.org/10.1023/A:1012734519752>
- Can, A., Megbolugbe, I., (1997). Spatial Dependence and House Price Index Construction. *Journal of Real Estate Finance and Economics*, 14 (1/2), 203–222. DOI: <http://dx.doi.org/10.1023/A:1007744706720>
- Chapman, J. I., Johnston, R. J., Tyrrell, T. J. (2009). Implications of a Land Value Tax with Error in Assessed Values. *Land Economics*, 85 (4), 576–586.
- Cichocinski, P. (2008). *Methods of Cartographical Presentation of Real Estate Values*. FIG Working Week, Stockholm, Sweden 14–19 June 2008.
- Clapp J., Rodriguez, M. (1998). Using a GIS for Real Estate Market Analysis: The Problem of Spatially Aggregated Data. *Journal of Real Estate Research*, 16 (1), 35–55.

- Colwell, P. F., Munneke, H. J. (1997). The Structure of Urban Land Prices. *Journal of Urban Economics*, 41 (3), 321–336. DOI: <http://dx.doi.org/10.1006/juec.1996.2000>
- Colwell, P. F., Munneke, H. J. (2003). Estimating a Price Surface for Vacant Land in an Urban Area. *Land Economics*, 79 (1), 15–28. DOI: <http://dx.doi.org/10.2307/31471101>
- Cotteleer, G., Gardebroeck, C., Luijt, J. (2008). Market Power in a GIS-Based Hedonic Price Model of Local Farmland Markets. *Land Economics*, 84 (4), 573–592.
- Earnhart, D. (2006). Using Contingent-Pricing Analysis to Value Open Space and Its Duration at Residential Locations. *Land Economics*, 82 (1), 17–35.
- Fik, T. J., Ling, D.C., Mulligan, G.F. (2003). Modeling Spatial Variation in Housing Prices: A Variable Interaction Approach. *Real Estate Economics*, 31 (4), 623–646. DOI: <http://dx.doi.org/10.1046/j.1080-8620.2003.00079.x>
- Fletcher, R. (1987). *Practical Methods of Optimization*. New York, John Wiley & Sons.
- Gall, J. (2006). Future of Value Maps in European Context. XXIII FIG Congress Munich, Germany, October 8-13.
- Galster, G., Tatian, P., Pettit, K. (2004). Supportive Housing and Neighborhood Property Value Externalities. *Land Economics*, 80 (1), 33–54. DOI: <http://dx.doi.org/10.2307/3147143>
- Guntermann, K. L., Thomas, G. (2005). Parcel Size, Location, and Commercial Land Values. *Journal of Real Estate Research*, 27 (3), 343–354.
- Hannoen, M. (2008). Predicting Urban Land Prices: A Comparison of Four Approaches. *International Journal of Strategic Property Management*, 12 (4), 217–236. DOI: <http://dx.doi.org/10.3846/1648-715X.2008.12.217-236>
- Haughwout, A., Orr, J., Bedoll, D. (2008). The Price of Land in the New York Metropolitan Area. Federal Reserve Bank of New York. *Current Issues in Economics and Finance*, 14 (3), 1–7.
- Hite, J. C., Terrell, E. J., Lu K. S. (1999). *Land Prices and the Changing Geography of Southern Row-Crop Agriculture*. Strom Thurmond Institute of Government and Public Affairs Clemson University, Southern Rural Development Center.
- Howes, C. K. (1980). *Value Maps: Aspects of Land and Property Values*. Norwich, Geobooks.
- Isaaks, E. H., Srivastava R. M. (1998). *Applied Geostatistics*, Oxford University, New York.
- Isakson, H. R. (1997). An Empirical Analysis of the Determinants of the Value of Vacant Land. *Journal of Real Estate Research*, 13 (2), 103–414.
- Kelley, P. R., Barry, R., Sirmans C. F. (1998). Spatial Statistics and Real Estate. *The Journal of Real Estate Finance and Economics*, 17 (1), 5–13. DOI: <http://dx.doi.org/10.1023/A:1007783811760>
- Kitanidis, P. K. (1994). Quasi-linear Geostatistical Theory for Inverting. *Water Resources Research*, 31 (10), 2411–2419. DOI: <http://dx.doi.org/10.1029/95WR01945>
- Kryobokov, M. (2004). Urban Land Zoning for Taxation Purposes in Ukraine. Possible methods under an Immature Land Market. *Property Management*, 22 (3), 214–229. DOI: <http://dx.doi.org/10.1108/02637470410545002>
- Liu, Y., Zheng, B., Huang, L., Tang, X. (2007). Urban Residential Land Value Analysis: Case Danyang, China. *Geo-spatial Information Science*, 10 (3), 228–234. DOI: <http://dx.doi.org/10.1007/s11806-007-0066-4>
- Maguire, D. J., Longley, P. A. (2005). The Emergence of Geoportals and their Role in Spatial Data Infrastructures. *Computers Environment and Urban Systems*, 29 (1), 3–14. DOI: <http://dx.doi.org/10.1016/j.compenvurbysys.2004.05.012>
- Matthews, J. W., Turnbull, G. K. (2007). Neighbourhood Street Layout and Property Value: The Interaction of Accessibility and Land Use Mix. *The Journal of Real Estate Finance and Economics*, 35 (2), 111–141. DOI: <http://dx.doi.org/10.1007/s11146-007-9035-9>
- McCluskey, W. J., Deddis, W. G., Lamont, I. G., Borst, R. A. (2000). The Application of Surface Generated Interpolation Models for the Prediction of Residential Property Values. *Journal of Property Investment & Finance*, 18 (2), 162–176. DOI: <http://dx.doi.org/10.1108/14635780010324321>
- Montero, J. M., Larraz, B. (2010). Estimating Housing Prices: A Proposal with Spatially Correlated Data. *International Advances in Economic Research*, 16 (1), 39–51. DOI: <http://dx.doi.org/10.1007/s11294-009-9244-5>
- Munroe, D. K. (2007). Exploring the Determinants of Spatial Pattern in Residential Land Markets: Amenities and Disamenities in Charlotte, NC, USA. *Environment and Planning B: Planning and Design*, 34 (2), 336–354. DOI: <http://dx.doi.org/10.1068/b32065>
- Noelwah, R. N. (2005). The Effect of Environmental Zoning and Amenities on Property Values: Portland, Oregon. *Land Economics*, 81 (2), 227–246.
- Páez, A. (2009). Recent research in spatial real estate hedonic analysis. *Journal of Geographical Systems*, 11 (4), 311–316. DOI: <http://dx.doi.org/10.1007/s10109-009-0103-y>
- Palmquist, R. B. (2005). Property Value Models. In *Handbook of Environmental Economics*, ed. K. G. Mäler and J. R. Vincent. North-Holland: Elsevier.
- Ping, A. (2005). Residential Land Value Modeling: case study of Hankou, China. Enschede, ITC.
- Schabenberger, O., Gotway, A. C. (2005). *Statistical Methods for Spatial Data Analysis*. Chapman & Hall/CRC Press.
- Tu, Y., H. Sun, Yu, S. (2007). Spatial Autocorrelations and Urban Housing Market Segmentation. *The Journal of Real Estate Finance and Economics*, 34 (3), 385–406. DOI: <http://dx.doi.org/10.1007/s11146-007-9015-0>
- Valente, J., Wu, S., Gelfand, A., Sirmans, C.F. (2005). Apartment Rent Prediction Using Spatial Modeling. *Journal of Real Estate Research*, 27 (1), 105–136.
- Vickers, T., Thurstain-Goodwin, M. (2002). Visualising Landvaluescape without a Cadastre. FIG XXII International Congress Washington, D.C. USA, April 19–26 2002.
- Walacki, M., Cellmer, R., Zrobek, S., (2013). Mass Appraisal – International Background, Polish Solutions and Proposal of new Methods Application. *Geodetski list*, 67 (4), 255–269.
- Waljiyanto, N. W., Suryohadi, W. (2004). Application of Finite Difference Method to Develop Land Value Map. 3rd FIG Regional Conference, Jakarta, Indonesia, October 3–4, 2004.
- Weiss, E. (2005). Replacing the 'Combined Tax on Land and Buildings' With a Simplified 'Land Value Tax' in the Federal Republic of Germany. *Geodetski Vestnik*, 49 (1), 29–34.
- Zrobek, S., Grzesik, C. (2013). Modern Challenges Facing the Valuation Profession and Allied University Education in Poland. *Real Estate Management and Valuation*. 21 (1), 14–18. DOI: <http://dx.doi.org/10.2478/remav-2013-0002>

Cellmer R., Belej M., Zrobek S., Šubic Kovač M. (2014). Urban land value maps – a methodological approach. *Geodetski vestnik*, 58 (3): 535-551. DOI: 10.15292/geodetski-vestnik.2014.03.535-551

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