ANALIZA ZANESLJIVOSTI MREŽE GNSS OB PRISOTNOSTI VEČ GROBIH POGREŠKOV MERITEV

ANALYSIS OF A GNSS NETWORK USING THE THEORY OF RELIABILITY FOR MULTIPLE OUTLIERS

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IZVLEČEK

Po tem, ko vzpostavimo geodetsko mrežo (horizontalno, nivelmansko, GNSS itd.), na podlagi geodetske izmere in z izravnavo po metodi najmanjših kvadratov določimo koordinate njenih točk. Morebitni grobi pogreški pri eni ali več meritvah vplivajo na vse nadaljnje izračune. Takšne meritve je zato treba odkriti in ustrezno popraviti. V praksi se največkrat izločijo iz izravnave in geodetska mreža se spet izravna. Za prepoznavanje grobo pogrešenih vrednosti meritev so na voljo različni statistični testi. Še več, ocene zanesljivosti rezultatov izravnave bi morale biti dodane rezultatom statističnih testov, s čimer bi pridobili več informacij o uspešnosti odkrivanja grobih pogreškov in njihovem vplivu na rezultate izravnave. Oceno zanesljivosti geodetske mreže je mogoče določiti za prisotnost le ene ali več grobo pogrešenih vrednosti meritev. V članku predstavljamo rezultate analize mreže GNSS, kjer so ocene zanesljivosti določene za prisotnost ene in več grobo pogrešenih vrednosti meritev. Rezultati kažejo, da se ocena zanesljivosti geodetske mreže poslabša, če je prisotnih več grobo pogrešenih vrednosti meritev.

ABSTRACT

After geodetic networks (e.g., horizontal control, leveling, GNSS etc.) are established, they are measured and point coordinates are estimated by the method of Least Squares. If one or more observations are burdened with errors, these contaminated observations affect the other good observations and may produce incorrect estimates of the parameters. Thus, these contaminated observations should be detected and corrected. Generally, in practice, they are removed and the network is readjusted. To detect the outliers among the observations statistical tests are performed. Yet, reliability measures should be accompanied to statistical tests to find out more about the ability of error detection and the effects of errors on the solutions. Now it is possible that reliability measures can be calculated for the cases of single or multiple outliers. In this paper, a GNSS network is analyzed using reliability measures in both cases. The results show that in the case of multiple outliers reliability measures worsen.

KLJUČNE BESEDE

mreža GNSS, grobi pogreški meritev, ocena zanesljivosti, več grobih pogreškov meritev

KEY WORDS

GNSS network, outliers, reliability measures, multiple outliers

1 INTRODUCTION

In spite of technological advancements and human efforts, it is impossible to obtain completely error free measurements. Thus, it is stated with absolute certainty that all measurements contain errors. Errors are classified into three categories as gross errors (blunders/outliers), systematic errors and random errors. Gross errors can be detected by repeating the measurements and systematic errors are eliminated by using calibrated instruments, applying corrections and following correct measurement procedures. Nevertheless, since random errors occur due to human and instrument imperfections, they cannot be removed. However, they conform to the laws of probability and follow the normal distribution theory. Hence, they are adjusted in a manner that follows these mathematical laws. The most commonly used adjustment method is called the Least Squares method.

Nowadays GNSS networks are widely used for various surveying purposes. As it is done with other geodetic networks, after measurements are taken the point coordinates in GNSS networks are estimated using the Least Squares method. Generally, the next step is outlier detection to find out whether there is any observation that is burdened with an error among the measurements. When an observation is burdened with an error it is called an outlier or an outlier observation. With almost all statistical tests it is assumed that observations are normally distributed. This means that gross errors and systematic errors have been removed and we are left with only random errors.

To detect the outliers among the measurements, statistical tests such as Baarda's method is often used. Nonetheless, Baarda's reliability theory is based on the assumption of a single outlier. On the other hand, multiple outliers are possible. Therefore, measures of reliability for multiple outliers must be used. It means that internal and external reliability measures have to be reformulated. A review of reliability measures for multiple outliers is given by Knight et al. (2010). They discussed that internal reliability measures for multiple outliers are equal or poorer than their corresponding values for a single outlier. In addition, they showed that the external reliability values are larger when multiple outliers are present. Thus, lower levels of internal and external reliability can be achieved when multiple outliers are considered.

First author of this paper considered multiple outliers in robustness analysis - see Yetkin and Berber (2013). Other recent studies on the subject: Baselga (2011) focused on the theory of outlier detection in least-squares adjustment and stated that although the case of a single outlier can be efficiently handled, extensions of the testing theory to the multiple outlier case seem questionable in rigor or applicability. Klein et al. (2015) evaluated, compared and discussed different methods for quality control in geodetic data analysis in the general scenario of correlated observations and multiple outliers.

In this paper, a GNSS network is analyzed using reliability measures in the case of single and multiple outliers.

2 RELIABILITY MEASURES

After measurements are taken the geodetic networks are adjusted using the Least Squares method. To detect the outliers in the data set, global test and outlier tests are used. Despite the use of these tests, one or more outliers might go undetected and they may produce incorrect estimates of the parameters. Since the number of outliers in the data set is unknown, in practice, an iterative approach is utilized to determine the most likely suspect of observations based on the assumption of a single outlier. However using the concept of the multiple correlation coefficients and by applying the Rayleigh-Ritz theorem, Knight et al (2010) generalized the measures of reliability to multiple outliers. These are outlined in the sequel.

2.1 Single outlier case

Baarda's approach is divided into internal reliability and external reliability. The maximum undetectable errors with controllability numbers and reliability numbers constitute the internal reliability of the network. The maximum undetectable errors among the observations which would not be detected by a statistical test are given by Baarda (1968) as

$$\Delta l_i = \sqrt{\frac{\lambda_0 \sigma_0^2}{\boldsymbol{h}_i^{\mathrm{T}} \mathbf{P} \mathbf{Q}_{\mathrm{v}} \mathbf{P} \boldsymbol{h}_i}} \quad i = 1, 2, \ldots, n \tag{1}$$

where n is the number of observations, **P** is the weight matrix; $\mathbf{Q}_v$ is the cofactor matrix of the estimated residuals; $\boldsymbol{h}_i$ is an n×1 vector, containing zeros with a one corresponding to *i*th observation; and σ_0^2 is the a-priori variance factor. λ_0 is the value of the shift (non-centrality parameter) of the postulated distribution in the alternative hypothesis. Needless to say that maximum undetectable errors are desired to be as small as possible.

The external reliability of a network that is computed for individual observations measures the effects of the undetected errors on the estimated parameters. The Least Squares estimate for the displacements $\Delta\mathbf{x}$ caused by the $\Delta\mathbf{l}$ in the observations is given by

$$\Delta\mathbf{x} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1} \mathbf{A}^{\mathrm{T}}\mathbf{P}\Delta\mathbf{l} \tag{2}$$

where **A** is the design matrix and $\Delta\mathbf{l}$ is a n×1 vector of zeros except for Δl_i in *i*th position.

Controllability is a measure of internal reliability that is derived from the maximum undetectable errors (minimum detectable biases). It is calculated as (Pelzer 1980, Wang and Chen 1994, Chen and Wang 1996 and Knight et al. 2010).

$$C_{0i} = \sqrt{\overline{\mathbf{h}_i^{\mathrm{T}} \mathbf{Q} \mathbf{h}_i \mathbf{h}_i^{\mathrm{T}} \mathbf{P} \mathbf{Q}_{\mathrm{v}} \mathbf{P} \mathbf{h}_i}} \tag{3}$$

where **Q** is cofactor matrix of the observations. Reliability numbers are derived from controllability and remove the effect of the non-centrality parameter $\sqrt{\lambda_0}$. For the correlated observations the reliability numbers are given as (Pelzer 1980, Wang and Chen 1994, Chen and Wang 1996 and Knight et al. 2010)

$$\overline{r_i} = \mathbf{h}_i^{\mathrm{T}} \mathbf{Q} \mathbf{h}_i \mathbf{h}_i^{\mathrm{T}} \mathbf{P} \mathbf{Q}_{\mathrm{v}} \mathbf{P} \mathbf{h}_i \tag{4}$$

The bounds for the redundancy numbers are

$$\mathbf{0} \le \overline{r_i} \le \mathbf{h}_i^{\mathrm{T}} \mathbf{Q} \mathbf{h}_i \mathbf{h}_i^{\mathrm{T}} \mathbf{P} \mathbf{h}_i \tag{5}$$

As well known, reliability numbers are desired to be as big as possible.

2.2 Multiple outlier case

The statistical tests used for outlier detection after least-squares adjustment is based on the assumption of a single outlier (Baarda 1968; Pope 1975) or multiple outliers (Kok 1984; Cross and Price 1985; Ding and Coleman 1996; Knight et al. 2010). Both approaches consist of a global model test and an outlier test. However, even these rigorous statistical tests do not guarantee 100% success for detection of all outliers; i.e., one or more outliers may not be detected. Therefore, the size of maximum undetectable error in a specific observation for a given non-centrality parameter $\sqrt{\lambda_0}$ must be computed to obtain a thorough examination of this undetected error's effect on the estimated parameters.

In both cases (global test and outlier test), the statistic follows a non-central chi-squared distribution. The non-central parameter of this distribution is given by

$$\lambda = \frac{\mathbf{z}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{Q}_{\mathrm{v}}\mathbf{P}\mathbf{H}\mathbf{z}}{\sigma_0^2} \tag{6}$$

where $\mathbf{z}$ is a vector of θ outliers and $\mathbf{H}$ is an $n\times\theta$ matrix, with rank θ, containing zeros with a one in each column corresponding to an outlier. Eq. (6) is a building block to detect the maximum undetectable errors that are determined based on the selected non-centrality parameter. The non-centrality parameter depends on the selection of Type I (α_0) and Type II (β_0) error levels for both global model test and outlier test. The degree of freedom and the number of outliers (single or multiple) also play an important role in non-centrality parameter of global model test and outlier test respectively. If we assume that the non-centrality parameter is equal and the error probabilities are appropriately selected for both tests, their maximum undetectable error vector is equal. This characteristic exists in the β-method (Knight et al. 2010). The β-method was used by Baarda (1968) for single outlier detection. On the other hand, it has been generalized by Kok (1984) for multiple outliers.

If there are multiple outliers, Eq. (6) does not provide an absolute solution for the maximum undetectable errors for a particular non-centrality parameter. Therefore, the maximum undetectable error in an observation when multiple outliers ($\theta > 1$) are considered is obtained using the Rayleigh-Ritz method. Similarly the maximum effect of undetected errors on a parameter when multiple outliers are considered can be obtained as follows. A mere summary is provided below; however, for detailed information, readers are referred to Knight et al. (2010).

If maximum undetectable error is generalized to multiple outliers the formula reads

$$\Delta l_i^{\theta} = \frac{\Delta l_i}{\sqrt{1-\wp^2_{\mathrm{H^T Pv\,i}}}} \tag{7}$$

where $\wp$ is the ith multiple correlation coefficient. For multiple outliers the controllability of the ith measurement for θ outliers C_{0i}^{θ} is

$$C_{0i}^{\theta} = \frac{C_{0i}}{\sqrt{1-\wp^2_{\mathrm{H^T Pvi}}}} \tag{8}$$

Similar to the single outlier case, reliability numbers can also be obtained for multiple outliers. The generalization of reliability numbers for θ outliers is

$$\overline{r}_i^{\theta} = \overline{r}_i\left(1-\wp^2_{\mathrm{H^T Pv}\,i}\right) \tag{9}$$

The bounds of reliability numbers are

$$0 \le \overline{r}_i^{\theta} \le \mathbf{h}_i^{\mathrm{T}}\mathbf{Q}\mathbf{h}_i\mathbf{h}_i^{\mathrm{T}}\mathbf{P}\mathbf{h}_i\left(1-\wp^2_{\mathbf{H^TPv}_i}\right) \tag{10}$$

One can infer that since the denominator in Eq. (7) varies between 0 and 1, Δl_i^{θ} is always larger than or equal to Δl_i. Similarly, C_{0i}^{θ} is always larger than or equal to C_{0i}. On the other hand, due to the same reasoning, $\overline{r}_i^{\theta}$ is always smaller than or equal to $\overline{r}_i$.

In order to derive the external reliability for multiple outliers the Rayleigh-Ritz method can be applied and this leads to an eigenvalue problem in which eigenvalues are given as:

$$\lambda = (\lambda_0\sigma_0^2\left(\mathbf{H}^{\mathrm{T}}\mathbf{PQ}_{\mathrm{v}}\mathbf{PH}\right)^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{PA}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\times c_t^{\mathrm{T}}c_t\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{PH}) \tag{11}$$

and the maximum external reliability of *j*th parameter y_{0j}^{θ} is

$$y_{0j}^{\theta} = \sqrt{\lambda_{max}} \quad j = 1, 2, \ldots, u \tag{12}$$

where u is the number of unknown parameters in the network. It should be noted that Eq. (12) can be used for single and multiple outliers; for instance, in this study, external reliability values are calculated for $\theta = 1$, $\theta = 2$ and $\theta = 3$.

3 NUMERICAL RESULTS

In this study, the described approach is tested on a real GNSS network that is established in the Treasure Coast Campus of Florida Atlantic University. The network is comprised of five points and eight baseline observations (see Fig. 1). The datum of the network is provided by minimum constraints, i.e., point 5 has been considered as a fixed station. The variance-covariance matrix for this GNSS network is a block-diagonal type with an individual 3 × 3 matrix for each measured baseline on the diagonal. All other elements of the matrix are zeros.

Figure 1: FAUNet (image from Google)

If it is assumed that there is at most one single outlier ($\theta = 1$) within the network, maximum undetectable error values, controllability values and reliability numbers can be obtained from Eqs. (1), (3) and (4). For $\sqrt{\lambda_0}$ =3.61 (for α_0=0.05 and β_0=0.05) the results in Table 1 are obtained. The external reliability values for a single outlier can also be obtained from Eq. (2). These results are given in Table 2. Since we have 24 observations in the network used, it was not possible to fit all the results for all measurements therefore only the results for measurements 1,2,3,4 and 24 are tabulated in Table 2.

Table 1: Internal reliability for single outlier.

i	σ_i (mm)	MDB_i (mm)	C_i	$\overline{r_i}$
1	0.2646	1.5978	6.0393	0.3572
2	0.5745	2.8823	5.0174	0.5176
3	0.3000	1.9480	6.4932	0.3090
4	0.3606	1.7235	4.7801	0.5703
5	0.7211	2.9456	4.0848	0.7809
6	0.4472	1.8374	4.1085	0.7719
7	0.3464	1.5966	4.6090	0.6134
8	0.7071	2.8048	3.9666	0.8281
9	0.4472	1.7717	3.9616	0.8303
10	0.3606	1.7444	4.8381	0.5567
11	0.8602	3.2333	3.7587	0.9223
12	0.7000	2.4509	3.5013	1.0629
13	0.3606	1.6592	4.6018	0.6153
14	0.8602	3.0849	3.5861	1.0132
15	0.7000	2.3147	3.3068	1.1916
16	0.2828	1.5774	5.5770	0.4189
17	0.7348	2.7291	3.7138	0.9447
18	0.5196	1.9296	3.7135	0.9449
19	0.2828	1.4950	5.2856	0.4664
20	0.7000	2.7217	3.8881	0.8619
21	0.5385	2.0032	3.7199	0.9416
22	0.3317	1.5551	4.6888	0.5927
23	0.6782	2.6672	3.9325	0.8426
24	0.4123	1.6906	4.1002	0.7751

Table 2: External reliability for single outlier (mm).

i	1	2	3	4		24
Δx_1	0.5327	0.0809	0.0411	0.4165	…	0.0212
Δy_1	0.1105	0.8617	0.0611	0.0535	…	0.0497
Δz_1	0.0389	0.0178	0.4478	0.0092	…	0.4406
Δx_2	0.5646	0.0654	0.0231	0.1866	…	0.0143
Δy_2	0.2987	1.0287	0.2881	0.1264	…	0.0645
Δz_2	0.2389	0.1270	1.2328	0.1007	…	0.4087

i	1	2	3	4		24
Δx_3	0.0905	0.0097	0.0192	0.1077	…	0.0067
Δy_3	0.0291	0.1807	0.0855	0.0263	…	0.0236
Δz_3	0.0580	0.0238	0.1956	0.0112	…	0.5377
Δx_4	0.0947	0.0084	0.0044	0.3812	…	0.0627
Δy_4	0.0037	0.1680	0.0228	0.0196	…	0.1295
Δz_4	0.0051	0.0238	0.0919	0.0179	…	0.9670

If reliability analysis is now considered for multiple outliers ($\theta = 2$), maximum undetectable bias values, controllability values and reliability numbers can be obtained from Eqs. (7), (8) and (9). These results are given in Table 3. Since there are 24 observations in the network, there are $\theta\binom{n}{\theta} = 2\binom{24}{2} = 552$ internal reliability results. Thus, to save some space only the results for the first measurement are given in the table. External reliability for two outliers can be obtained from Eq. (12) and they are shown in Table 4. For external reliability there are $\binom{n}{\theta} = \binom{24}{2} = 276$ external reliability vectors. Again because the table is too big to fit to a page, only the results for 1-2, 1-3, 1-4 and 1-24 are displayed.

Table 3: Internal reliability for multiple outliers.

i-j	MDB_i (mm)	C_i^{θ}	r_i^{θ}
1-2	1.6060	6.0701	0.3536
1-3	1.6108	6.0881	0.3515
1-4	1.8024	6.8124	0.2808
1-5	1.6003	6.0486	0.3562
1-6	1.5993	6.0446	0.3566
1-7	1.9456	7.3538	0.2409
1-8	1.5995	6.0455	0.3565
1-9	1.6023	6.0561	0.3553
1-10	1.8421	6.9625	0.2688
1-11	1.6009	6.0507	0.3559
1-12	1.6010	6.0512	0.3558
1-13	1.9367	7.3201	0.2432
1-14	1.5998	6.0466	0.3564
1-15	1.6045	6.0644	0.3543
1-16	1.6803	6.3511	0.3230
1-17	1.6011	6.0517	0.3558
1-18	1.5979	6.0394	0.3572
1-19	1.6122	6.0936	0.3509
1-20	1.5980	6.0397	0.3572
1-21	1.5994	6.0452	0.3565

i-j	MDB_i (mm)	C_i^θ	r_i^θ
1-22	1.6080	6.0778	0.3527
1-23	1.5980	6.0398	0.3572
1-24	1.5987	6.0425	0.3569

Table 4. External reliability for multiple outliers (mm).

i	1-2	1-3	1-4		1-24
Δx_1	0.5334	0.5438	0.9182	…	0.5341
Δy_1	0.8620	0.1339	0.1105	…	0.1227
Δz_1	0.0446	0.4580	0.0401	…	0.4413
Δx_2	0.5647	0.5667	0.5708	…	0.5646
Δy_2	1.0472	0.3910	0.2990	…	0.3037
Δz_2	0.2830	1.2356	0.2391	…	0.4669
Δx_3	0.0925	0.0909	0.1915	…	0.0906
Δy_3	0.1810	0.0875	0.0534	…	0.0369
Δz_3	0.0608	0.1985	0.0606	…	0.5393
Δx_4	0.0964	0.0950	0.3921	…	0.1154
Δy_4	0.1692	0.0238	0.0205	…	0.1295
Δz_4	0.0249	0.0934	0.0234	…	0.9674

Internal and external reliability measures for three outliers ($\theta = 3$) are given in Tables 5 and 6 respectively. Since there are 24 observations in the network, there are $\theta\binom{n}{\theta} = 3\binom{24}{3} = 6072$ internal reliability results and $\binom{n}{\theta} = \binom{24}{3} = 2024$ external reliability vectors.

Table 5: Internal reliability for multiple outliers.

i-j	MDB_i (mm)	C_i^θ	r_i^θ
1-2-3	1.6508	6.2395	0.3347
1-2-4	1.8089	6.8369	0.2788
1-2-5	1.6061	6.0705	0.3536
1-2-6	1.6101	6.0858	0.3518
1-2-7	1.9533	7.3830	0.2390
1-2-8	1.6061	6.0705	0.3536
1-2-9	1.6154	6.1056	0.3495
1-2-10	1.8483	6.9858	0.2670
1-2-11	1.6061	6.0705	0.3536
1-2-12	1.6150	6.1041	0.3497
1-2-13	1.9446	7.3499	0.2412
1-2-14	1.6061	6.0705	0.3536
1-2-15	1.6217	6.1294	0.3468

i-j	MDB$_i$ (mm)	C_i^θ	r_i^θ
1-2-16	1.6864	6.3738	0.3207
1-2-17	1.6069	6.0736	0.3532
1-2-18	1.6067	6.0728	0.3533
1-2-19	1.6208	6.1262	0.3472
1-2-20	1.6065	6.0719	0.3534
1-2-21	1.6081	6.0779	0.3527
1-2-22	1.6167	6.1104	0.3490
1-2-23	1.6064	6.0718	0.3534
1-2-24	1.6072	6.0746	0.3531

Table 6: External reliability for multiple outliers (mm).

i	1-2-3	1-2-4	1-3-4		1-2-24
Δx_1	0.5460	0.9188	0.9253	…	0.5347
Δy_1	1.0301	0.8620	0.1339	…	0.8620
Δz_1	0.5949	0.0456	0.4582	…	0.4413
Δx_2	0.5673	0.5709	0.5730	…	0.5647
Δy_2	1.5137	1.0475	0.3912	…	1.0530
Δz_2	1.4421	0.2832	1.2357	…	0.4846
Δx_3	0.0926	0.1922	0.1916	…	0.0925
Δy_3	0.3022	0.1869	0.0981	…	0.1839
Δz_3	0.2608	0.0633	0.1991	…	0.5414
Δx_4	0.0993	0.3928	0.3922	…	0.1174
Δy_4	0.2330	0.1706	0.0311	…	0.2076
Δz_4	0.1389	0.0336	0.0963	…	0.9678

In order to be able to compare internal reliability measures for multiple outliers ($\theta = 2$) against single outliers, each row from Table 3 should be compared against the corresponding two rows in Table 1. For example, the row 1-2 in Table 3 should be compared against the rows 1 and 2 in Table 1, the row 1-3 in Table 3 should be compared against the rows 1 and 3 in Table 1 and so on. Similarly, for three outliers ($\theta = 3$) case, the row contains measurements 1-2-3 in Table 5 should be compared against the rows 1-2 and 1-3 in Table 3, the row contains measurements 1-2-4 in Table 5 should be compared against the rows 1-2 and 1-4 in Table 3 and so on.

Comparisons for external reliability measures for multiple outliers are done similarly meaning that each column from Table 4 should be compared against the corresponding two columns in Table 2. For example, the column 1-2 in Table 4 should be compared against the columns 1 and 2 in Table 2, the column 1-3 should be compared against the columns 1 and 3 and so on. For three outliers ($\theta = 3$) case, the column 1-2-3 in Table 6 should be compared against the columns 1-2 and 1-3 in Table 4, the column 1-2-4 should be compared against the columns 1-2 and 1-4 and so on.

As can be seen from Table 3, all of the maximum undetectable errors and the controllability numbers are greater than the single outlier values of Table 1. In addition, the reliability numbers for multiple outliers are smaller than their single outlier counterparts. If the values in Table 5 are compared against the values in Table 3, the same pattern appears.

External reliability values are larger in Table 4 than the values for single outlier case in Table 2 and the external reliability values are larger in Table 6 ($\theta = 3$) than the values in Table 4 (for $\theta = 2$).

4 CONCLUSIONS

To ponder over the commonly used reliability measures, reliability measures for multiple outliers ought to be utilized. In this case, internal and external reliability measures are reformulated. In addition, use of these reliability measures should be accompanied to statistical tests to find out more about the ability of error detection and the effects of errors on the solutions. In this paper, a real network (GNSS network) is analyzed first time using reliability measures for multiple outliers. The results show that in the case of multiple outliers internal reliability measures i.e., maximum undetectable errors and the controllability numbers are greater than their single outlier counterparts. In addition, the reliability numbers for multiple outliers are smaller than their single outlier counterparts. In the case of multiple outliers external reliability measures are larger than the values for single outlier case.

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