

KOTNA IZRAVNAVA IZMERJENEGA POLIGONA STAVBE Z ROBUSTNO METODO M-OCENJEVANJA

ANGULAR ADJUSTMENT OF SURVEYED BUILDING POLYGON USING ROBUST M-ESTIMATION METHODS

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IZVLEČEK

Položaji točk poligona, ki določajo geometrično obliko stavbe, so pogojeni z naključnimi merskimi napakami kotov geometrije objekta. Če tako primerjamo dejanske in načrtovane kote poligona stavbe, ugotovimo, da niso enaki. Izmerjene vrednosti kotov poligona stavbe je mogoče popraviti v okviru standardnega odklona položaja poligonskih točk in s tem uskladiti vrednosti merjenih kotov z načrtovanimi. Za izravnavo kotov poligona je predvidena izravnavo po metodi najmanjših kvadratov robustno glede na kote poligona, ki najbolj odstopajo od načrtovanih vrednosti. Kot rezultat izravnave dobimo kote, ki ustrezajo načrtovanim vrednostim, hkrati pa prepoznamo kote, ki značilno odstopajo od načrtovanih vrednosti, in sicer skupaj z ocenjenim odstopanjem. Pri tem postopku se nekoliko spremeni vrednost koordinat poligonskih točk, vendar v mejah njihovega standardnega odklona.

ABSTRACT

Positions of polygon points determining a geometric shape of a building object are affected by random measurement errors of the object's angles. Polygon angles are thus not equal when comparing actual and design corners of a building object. However it is possible to make an attempt to correct polygon angles within the standard deviation of the position of polygon points and lead to their congruency with the known design angles of the field object. In this paper, we propose the least squares method for angular adjustment of a polygon which is robust to angles much outlying from their design values. As a result the adjusted polygon has angles equal to the design angles and also detected angles outlying from the design ones, with determined values of deviations. Meanwhile, the coordinates of polygon points slightly changed within their standard deviations.

KLJUČNE BESEDE

prostorska podatkovna baza, stavba, geometrija objekta, izravnavo poligona objekta, robustna ocena

KEY WORDS

geospatial database, building object geometry, adjustment of the polygon object, robust estimation

1 INTRODUCTION

The polygon geometry of a building object is described by consecutive points which determine the geometry of an object (Burrough, 1998; Longley et al., 2006; O' Sullivan, 2006). The coordinates of these points are obtained from direct field measurements as well as indirect measurements, such as airborne, satellite and drone photogrammetry and also through digitization of paper maps (Ghilani and Wolf, 2012; Wei, 2014; Mendela-Anzlik and Borkowski, 2017). These coordinates are affected by random measurement errors. The angles that are computed on the basis of the coordinates of a polygon object are also affected by random position errors of the points. The angles of a polygon object are thus not equal when comparing actual and design angles of a field object.

These angles can be however corrected within the standard deviations of the position of points and lead to their congruency with the known design angles of a field object. In this paper, we propose the least squares method for angular adjustment of a polygon which is robust to angles much outlying from their design values. As a result the adjusted polygon has angles equal to the design angles and also detected angles outlying from the design ones. Meanwhile, the positions of polygon points slightly move within their standard deviations. A similar solution is applied in a certain new method of a direct field measurement of building objects based on GNSS linear network, localized on the measured object (Osada, Karsznia K. and Karsznia I., 2018). The robust estimation methods have a wide range of applications in geodesy and surveying, e.g. (Mąkowski and Osada, 2005; Wu, Qiu and Wang, 2005; Banaś, 2012; Muszyński, Zienkiewicz and Baryła, 2015; Adamczyk, 2017; Akram, Liu and Qian, 2018), while building polygon adjustment in a geographic information system (GIS) is not widely discussed in the literature. However, it is worth to mention about an interesting approach, presented in the paper (Jin, Tong and Zhang, 2018). There is proposed the partial total-least-squares adjustment method for condition equations (PTLSC), which allows for adjustment of the interior angles of the digitized buildings to right angles as well as maintenance of the correlations among the elements in the observation vector and the coefficient matrix.

Concerning geospatial databases, the identified building shape often need to be generalized and regularized with the use of appropriate GIS tools (Burdeos, Makinano-Santillan and Amora, 2015; Mendela-Anzlik, 2015) or other approaches dedicated for buildings of both simple and more complicated geometry (Jones and Ware, 2005; Zhang and Stoter, 2013; Lokhat and Touya, 2016). However, none of them assumes the standard deviation of the position of the points as one of the parameters in a computational process. Though it is possible to use Python modules, such as SciPy or NumPy, integrated with GIS software (e.g. ArcGIS), for implementation of the appropriate algorithms that can take into account this positional accuracy parameter. Additionally, some solutions concerning the geometry of rectangles (Preparata and Shamos, 1985) could be also useful for the above mentioned task. As far as geospatial databases are concerned, there is still a need to come up with new ideas and also develop the existing solutions for angular adjustment of building polygons. Thus, the proposed method can be used for obtaining the geometrical shape of corrected buildings polygons as objects of geospatial databases and also as a final stage of situational measurement of a building object, before giving a database to the Office for Geodetic and Cartographic Documentation.

2 THE PROPOSED METHOD

The polygonal geometry of a building is specified by measured coordinates of the building corners $P_i(x_i, y_i)$, $i = 1, 2, \dots, n$ (Figure 1). The angles of the polygon at the points P_i can be computed from their coordinates according to Ogundare (2015) and Ghilani (2018), (Figure 1):

$$\alpha_i = \arctg\left(\frac{y_{i-1} - y_i}{x_{i-1} - x_i}\right) - \arctg\left(\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right) \tag{1}$$

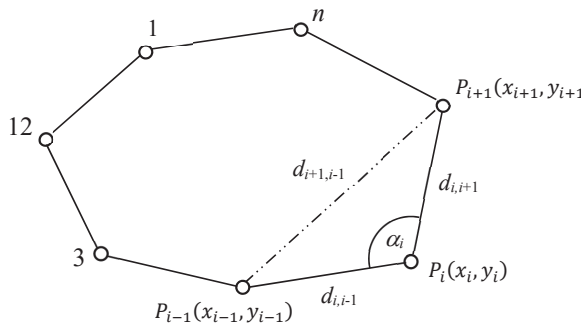


Figure 1: The polygonal geometry of a building.

The accuracy of the position of the points P_i is determined by their standard deviations (Ogundare, 2015):

$$\sigma_{P_i} = \sqrt{\sigma_{x_i}^2 + \sigma_{y_i}^2} \tag{2}$$

where σ_{x_i} , σ_{y_i} are standard deviations of the measured coordinates x_i , y_i .

If σ_{x_i} , σ_{y_i} are unknown and σ_{P_i} is known then the standard deviations of the coordinates are usually computed assuming

$$\begin{aligned} \sigma_{x_i} &= \sigma_{y_i}, \tag{2} \\ \sigma_{x_i} = \sigma_{y_i} &= \sigma_{P_i} / \sqrt{2} \tag{3} \end{aligned}$$

The standard deviations σ_{α_i} of the angles α_i are computed from (1) based on the random error propagation law (Ogundare, 2015; Ghilani, 2018). In the simplest case assuming equal errors σ_{P_i} of the points P_i and equal errors of their coordinates $\sigma_{x_i} = \sigma_{y_i}$ (3) it is obtained:

$$\sigma_{\alpha_i} = \frac{\sigma_{P_i} \sqrt{d_{i,i+1}^2 + d_{i,i-1}^2 + d_{i+1,i-1}^2}}{\sqrt{2} d_{i,i+1} d_{i,i-1}} \tag{4}$$

where $d_{i,i-1}$, $d_{i,i+1}$, $d_{i+1,i-1}$ are distances between points (P_i, P_{i-1}) , (P_i, P_{i+1}) , (P_{i+1}, P_{i-1}) of the polygon (Figure 1).

The computed angles of the polygon $\alpha_i \pm \sigma_{\alpha_i}$ (1) (4) are not equal to the actual as well as to the theoretical design angles β_i of the building, taken from the existing building geometry by default.

The angles α_i are functions of the coordinates of three points P_{i-1} , P_i , P_{i+1} (1), (Figure 1). Their small differential changes $d\alpha_i$ are connected with the small differential changes $(v_{x_{i-1}}, v_{y_{i-1}})$, (v_x, v_y) , $(v_{x_{i+1}}, v_{y_{i+1}})$ of the coordinates of the points (x_{i-1}, y_{i-1}) , (x_i, y_i) , (x_{i+1}, y_{i+1}) , according to the differentials:

$$d\alpha_i = -\left(\frac{y_{i-1} - y_i}{d_{i,i-1}^2} - \frac{y_{i+1} - y_i}{d_{i,i+1}^2}\right)v_{x_i} + \left(\frac{x_{i-1} - x_i}{d_{i,i-1}^2} - \frac{x_{i+1} - x_i}{d_{i,i+1}^2}\right)v_{y_i} + \frac{y_{i-1} - y_i}{d_{i,i-1}^2}v_{x_{i-1}} - \frac{x_{i-1} - x_i}{d_{i,i-1}^2}v_{y_{i-1}} - \frac{y_{i+1} - y_i}{d_{i,i+1}^2}v_{x_{i+1}} + \frac{x_{i+1} - x_i}{d_{i,i+1}^2}v_{y_{i+1}} \tag{5}$$

So, the theoretical design angles β_i can be connected with the computed polygon angles α_i (1) according to the formula:

$$\beta_i + v_{\beta_i} = \alpha_i + d\alpha_i \tag{6}$$

where the differentials $d\alpha_i$ are given by (5) and v_{β_i} denote assumed small random corrections of the angles β_i with zero value expectation and standard deviation σ_{β_i} .

Fitting the theoretical design polygon angles ($\beta_1, \beta_2, \dots, \beta_n$) to the points (P_1, P_2, \dots, P_n) can be implemented by the weighted least squares method as the result of minimization of the standardized residual corrections angles and coordinates:

$$\sum_{i=1}^n \left\{ \left(\frac{v_{\beta_i}}{\sigma_{\beta_i}} \right)^2 + \left(\frac{v_{x_i}}{\sigma_{x_i}} \right)^2 + \left(\frac{v_{y_i}}{\sigma_{y_i}} \right)^2 \right\} = \min \quad \square \tag{7}$$

The set of observational equations (6) composed for all polygon angles ($\beta_1, \beta_2, \dots, \beta_n$) has a well-known form as the condition adjustment model (Leick et al., 2015): $\mathbf{B}\mathbf{v} + \mathbf{w} = \mathbf{0}$, where $\mathbf{v} = (v_{\beta_1}, v_{x_1}, v_{y_1}, \dots, v_{\beta_n}, v_{x_n}, v_{y_n})^T$ is vector of all unknown corrections, while the coefficients matrix \mathbf{B} and vector $\mathbf{w} = (\beta_1 - \alpha_1, \beta_2 - \alpha_2, \dots, \beta_n - \alpha_n)^T$ are known. It is solved under condition (7) or in the matrix notation $\mathbf{v}^T \Sigma^{-1} \mathbf{v} = \min$, where Σ is diagonal covariance matrix of the angles and coordinates $\text{diag}(\Sigma) = (\sigma_{\beta_1}^2, \sigma_{x_1}^2, \sigma_{y_1}^2, \dots, \sigma_{\beta_n}^2, \sigma_{x_n}^2, \sigma_{y_n}^2)$. The solution is given by (Leick et al., 2015): $\mathbf{v} = \mathbf{P}^{-1} \mathbf{B}^T (\mathbf{B} \mathbf{P}^{-1} \mathbf{B}^T)^{-1} \mathbf{w}$, where $\mathbf{P} = \Sigma^{-1}$.

The adjustment of the polygon starts from the computed standard deviations of coordinates $\sigma_{x_i}, \sigma_{y_i}$ (3), assuming that the design angles β_i are error-free $\sigma_{\beta_i} = 0$. The ratio $r = |v|/\sigma_v$, ($r_\beta = |v_\beta|/\sigma_{v_\beta}$, $r_x = |v_x|/\sigma_{v_x}$, $r_y = |v_y|/\sigma_{v_y}$) of the random corrections of observations v to their standard deviations σ_v is tested: $r \leq c$ (slightly). The parameter c defines the range of acceptable random errors $|v| \leq c\sigma_v$, usually $c = 3$. The so-called three sigma error $|v| \leq 3\sigma_v$ is often used as a deterministic criterion for rejecting individual observations from sets of data (Ghilani and Wolf, 2012, Osada et al. 2017).

As a result, the location of the adjusted polygon is affected by random errors of the coordinates of the polygon points whereas the design polygon angles ($\beta_1, \beta_2, \dots, \beta_n$) are preserved. The results of the adjustment will not be correct ($r > c$), Figure 2, if the real shape of the object deviates from the design shape. In such a case, in order to detect the outlying angles from their theoretical design values ($\beta_1, \beta_2, \dots, \beta_n$) it is proposed to use the adjustment of the polygon that is robust to outlying angles (Figure 2). In this robust adjustment, the angles ($\beta_1, \beta_2, \dots, \beta_n$) are considered as measured with very small errors in the first step of the iteration. In this case, the design shape of the polygon is kept at the points for which there is no physical deformation of the object's shape, while angular deformations are detected at the remaining points. The detected congruent angles usually obtain very small random corrections v . Hence, in the final adjustment of the polygon with conditions on the detected congruent angles (Figure 2), these angles are treated as error-free. Finally the adjusted polygon (P_1, P_2, \dots, P_n) contains new coordinates $x_i + v_{x_i}, y_i + v_{y_i}$ and new angles $\beta_i + v_{\beta_i}$. The angles for which $v_{\beta_i} = 0$ are detected as equal to the theoretical

design angles β_p , whereas the angles for which $v_{\beta_i} \neq 0$ are detected as outlying from the theoretical design angles β_p , with the deviations equal to v_{β_i} :

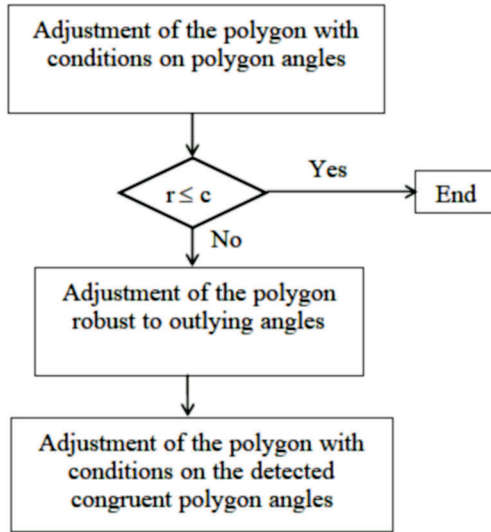


Figure 2: Flowchart of the proposed method.

3 TEST OF THE METHOD

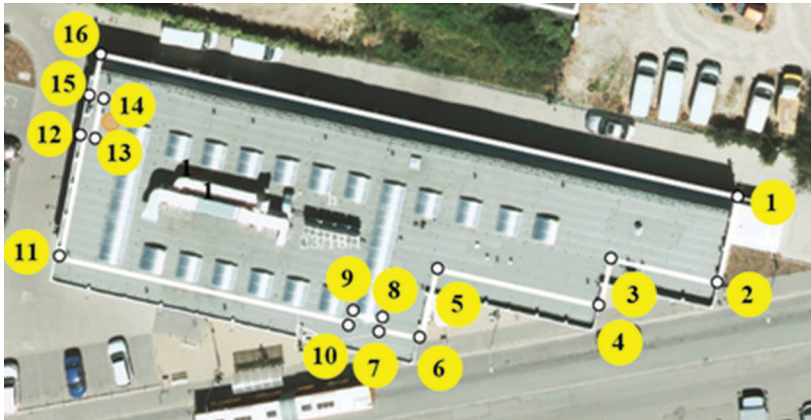


Figure 3: The polygon points 1, 2, ... 16 of the test object (51° 08' 33.2", 17° 07' 40.8")

The test of the proposed method was carried out on the example of a building located in Wrocław, Poland (Figure 3). The coordinates x, y of the corners 1, 2, 3, ..., 16 (Figure 3) are given in Table 1. The known theoretical design angles β_i and their corresponding polygon angles α_i (1) as well as their differences $\beta_i - \alpha_i$ are given in Table 2. The standard deviation of the position of the points is equal to $\sigma_{p_j} = 0.010$ m, whence $\sigma_{x_j} = \sigma_{y_j} = 0.0071$ m (3). The computed standard deviations of the angles σ_{a_i} (4) are bigger than the angle differences $\beta_i - \alpha_i$ at the points 1, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15 (Table 2). It means that probably at this thirteen points the polygon angles α_i can be replaced by the theoretical design angles β_i ,

At points 2, 11 and 16 the standard deviations of the angles σ_{α_i} are smaller than the differences $\beta_i - \alpha_i$. It means that probably at these three points the polygon angles α_i cannot be replaced by the theoretical design angles β_i , the polygon angles can be much outlying from the design angles.

4 RESULTS

4.1 Adjustment of the polygon with conditions on polygon angles

The adjustment of the polygon considering error-free conditional angles ($\sigma_\beta = 0$ grad), (Figure 2) leads to a significant deformation of the polygon $\max(r_x, r_y) = 37.313$ and the a posteriori variance of unit weight $\sqrt{\frac{\mathbf{v}^T \mathbf{P} \mathbf{v}}{\text{rows}(\mathbf{B})}} = 9.788$ that significantly exceeds the expected value $\sigma_0 = 1$ (Table 2). This result sug-

gests the existence of a physical non-perpendicularity of some of the building sides. In the following chapter (4.2), it is proposed the adjustment robust to outlying angles which makes it possible to detect the outlying angles from their theoretical projected values.

4.2 Adjustment of the polygon robust to outlying angles

Further adjustment of the polygon with conditions on the object angles (Figure 2) is carried out iteratively. At every step the weights p_β of the angles β are modified $p_\beta \leftarrow p_\beta f(v_\beta)$ using a weight function $f(v_\alpha)$, for example:

1) Huber weight function (Huber, 1981):

$$f(v_\alpha) = \begin{cases} 1 & |v_\alpha| \leq r\sigma_\alpha \\ \frac{r\sigma_\alpha}{|v_\alpha|} & |v_\alpha| > r\sigma_\alpha \end{cases} \tag{8}$$

where $r = 1.5$ (Erenoglu and Hekimoglu, 2009).

2) Huber modified weight function (Osada et al., 2017):

$$f(v_\alpha) = \begin{cases} 1 & |v_\alpha| \leq r\sigma_\alpha \\ \frac{1}{\left(1 + \frac{|v_\alpha|}{\sigma_\alpha} - r\right)^2} & |v_\alpha| > r\sigma_\alpha \end{cases} \tag{9}$$

where $r = 1.5$.

3) Hampel weight function (Hampel et al., 1986):

$$f(v_\alpha) = \begin{cases} 1 & |v_\alpha| \leq a\sigma_\alpha \\ \frac{a\sigma_\alpha}{|v_\alpha|} & a\sigma_\alpha < |v_\alpha| \leq b\sigma_\alpha \\ \frac{c\sigma_\alpha - |v_\alpha|}{(c-b)|v_\alpha|} & b\sigma_\alpha < |v_\alpha| \leq c\sigma_\alpha \\ 0 & |v_\alpha| > c\sigma_\alpha \end{cases} \tag{10}$$

where $a = 1.5$, $b = 3$ and $c = 6$ (Erenoglu and Hekimoglu, 2009).

4) Krarup weight function (Erenoglu and Hekimoglu, 2009):

$$f(v_\alpha) = \begin{cases} 1 & |v_\alpha| \leq r\sigma_\alpha \\ \exp\left(-\frac{|v_\alpha|}{r\sigma_\alpha}\right) & |v_\alpha| > r\sigma_\alpha \end{cases} \quad (11)$$

where $r = 3$.

5) Kraus weight function (Kraus, 2000):

$$f(v_\alpha) = \begin{cases} 1 & |v_\alpha| \leq r\sigma_\alpha \\ \frac{1}{1 + \left(\frac{|v_\alpha|}{\sigma_\alpha}\right)^c} & |v_\alpha| > r\sigma_\alpha \end{cases} \quad (12)$$

where a, c are empirically selected parameters.

6) Yang weight function (Yang et al., 1999),

$$f(v_\alpha) = \begin{cases} 1 & |v_\alpha| \leq a\sigma_\alpha \\ \frac{a\sigma_\alpha}{|v_\alpha|} \left(\frac{b - \frac{|v_\alpha|}{\sigma_\alpha}}{b - a} \right)^2 & a\sigma_\alpha < |v_\alpha| \leq b\sigma_\alpha \\ 0 & |v_\alpha| > b\sigma_\alpha \end{cases} \quad (13)$$

where a and b are chosen as 1.0 – 1.5 and 3.0 – 6.0, respectively.

In the case of the weight function (9) starting from initial small values of the angular standard deviation $\sigma_\beta = 0.0005, 0.0010, 0.0015 \dots$ grad after a few iterations, the adjustment process has stabilized at the acceptable level of $\sigma_0 = 0.995$ for $\sigma_\beta = 0.0020$ grad (Table 2). Finally only 2 outlying angles at the points 11 and 16 are detected, their corrections v_β are equal to 1.2252 grad and -1.2245, respectively (Table 2). All other 18 angles obtain very small random corrections $\max(v_\beta) = 0.0002$ grad (Table 2). Hence, in the final adjustment (Figure 2) these congruent angles are treated as error-free ($\sigma_\beta = 0$ grad).

The results of the detection of the outlying angles using all of the weight functions (8)–(13) are practically the same (Table 2). The modified Huber (9), Hampel (10), Krarup (11) and Kraus (12) functions give also very small, almost completely identical corrections at all other detected congruent angles (Table 2).

4.3 Adjustment of the polygon with conditions on the detected congruent polygon angles

Finally, introducing zero-mean-error values for detected congruent angles to design angles $\sigma_\beta = 0$ grad, and $\sigma_\beta = 10$ grad for two detected outlying angles at the points 11 and 16, the adjusted polygon is not deformed: , (Table 2). The adjusted polygon contains the theoretical design angles β_i at the fourteen points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, for which the corrections v_{β_i} are equal to zero, $v_{\beta_i} = 0$ (Table 2). Only at the two points, 11 and 16, the angles are detected as outlying from the theoretical

design angles β_{11}, β_{16} (Table 2). The deviations of the angles with respect to the theoretical design angles are $v_{\beta_{11}} = 1.2257$ grad and $v_{\beta_{16}} = -1.2257$ grad (Table 2). The coordinate differences $\Delta x_p, \Delta y_i$ of the adjusted positions of the polygon points $x_i + v_{x_i}, y_i + v_{y_i}$ and the known database positions of these points x_p, y_i do not exceed two times of the starting standard deviation values of the coordinates $\sigma_{x_j} = \sigma_{y_j} = 0.0071$ m (Table 1).

Table 1: The measured and adjusted positions of the polygon points [m]

Point	Measured positions		Adjusted positions		Differences	
	x	y	$x + v_x$	$y + v_y$	Δx	Δy
1	7866.422	9011.471	7866.422	9011.469	0.000	-0.002
2	7857.797	9009.556	7857.783	9009.555	-0.014	-0.001
3	7860.151	8998.804	7860.163	8998.814	0.012	0.010
4	7855.610	8997.812	7855.616	8997.806	0.006	-0.006
5	7859.228	8981.528	7859.223	8981.528	-0.005	0.000
6	7852.404	8980.020	7852.402	8980.017	-0.002	-0.003
7	7853.298	8975.950	7853.304	8975.948	0.006	-0.002
8	7854.147	8976.131	7854.149	8976.136	0.002	0.005
9	7854.818	8973.129	7854.815	8973.129	-0.003	0.000
10	7853.970	8972.942	7853.970	8972.941	0.000	-0.001
11	7860.371	8944.064	7860.368	8944.060	-0.003	-0.004
12	7872.500	8946.500	7872.496	8946.503	-0.004	0.003
13	7872.305	8947.441	7872.308	8947.439	0.003	-0.002
14	7876.298	8948.241	7876.300	8948.243	0.002	0.002
15	7876.491	8947.300	7876.488	8947.307	-0.003	0.007
16	7880.458	8948.116	7880.460	8948.107	0.002	-0.009

Table 2: The polygon angles corrections

The angles		Values of the angles [grad]												
Vertex	Left arm	Right arm	Theoretical β	Computed from coordinates α	Differences $\beta - \alpha$	Standard deviation σ_α	Adjusted with conditions on object angles	Adjustment of the polygon robust to outlying angles					Adjusted with detected congruent angles conditions	
								Huber modified	Hampel	Krarpup	Krauss	Yang		Huber
1	2	16	100	99.9706	0.0294	0.1454	100	100	100	100	100.0002	99.9909	100	
2	3	1	100	100.1878	-0.1878	0.1848	100	100	100	100	100.0003	100.1524	100	
3	4	2	300	300.0293	-0.0293	0.2974	300	299.9998	299.9997	299.9998	299.9998	299.9992	299.9803	300
4	5	3	100	99.7737	0.2263	0.2844	100	99.9998	99.9997	99.9998	99.9998	99.9989	99.8449	100
5	6	4	300	300.0726	-0.0726	0.1975	300	299.9999	299.9998	299.9999	299.9999	299.9996	300.0578	300
6	7	5	100	100.0807	-0.0807	0.3557	100	99.9999	99.9998	99.9999	99.9999	99.9995	100.0232	100
7	8	6	100	100.3931	-0.3931	1.4969	100	100	100	100	100	99.9999	99.9756	100
7	11	6	200	199.8685	0.1315	0.3080	200	199.9999	199.9998	199.9999	199.9999	199.9995	199.9393	200
8	9	7	300	299.3726	0.6274	1.5228	300	300	300	300	300	299.9999	299.9398	300
9	10	8	300	300.1819	-0.1819	1.5229	300	300	300	300	300	299.9999	300.0187	300
10	11	9	100	99.9309	0.0691	1.4662	100	100	100	100	100	99.9999	100.0128	100
10	11	6	200	199.9753	0.0247	0.1808	200	200	200	200	200	200	200.0032	200
11	12	10	100	101.2685	-1.2685	0.1116	100	101.2252	101.2266	101.2267	101.2252	101.2195	101.2301	101.2257
12	13	11	100	99.6097	0.3903	1.3295	100	100	100	100	100	100	99.9806	100
12	16	11	200	199.8639	0.1361	0.1875	200	200	200	200	200	199.9999	199.9333	200
13	14	12	300	300.4203	-0.4203	1.3619	300	300	300	300	300	300	300.0114	300
14	15	13	300	299.7095	0.2905	1.3620	300	300	300	300	300	299.9999	299.892	300
15	16	14	100	99.9636	0.0364	1.3624	100	100	100	100	100	99.9999	99.9019	100
15	16	11	200	199.6972	0.3028	0.3237	200	200	200	200	200	199.9998	199.7661	200
16	1	15	100	99.0351	0.9649	0.3150	100	98.7755	98.7745	98.7740	98.7755	98.7834	98.9879	98.7743
The general quality indicators														
Starting value σ_α [grad]							10 ⁻⁶	0.0020	0.0025	0.0020	0.0020	0.0050	0.2000	10 ^{-6,2)}
σ_0							9.788	0.955	0.913	0.900	0.954	1.184	1.064	0.903
$\max(r_x, r_y)$							37.313	2.776	2.776	2.778	2.776	2.195	2.353	2.775
$\max(r_\alpha)$							140.27	0.910	0.914	0.916	0.910	1.285	6.359	0.963

¹⁾ the angles are computed from adjusted coordinates

²⁾ $\sigma_\alpha = 10^{-6}$ grad for detected congruent 18 angles and $\sigma_\alpha = 10$ grad for detected 2 outlying angles

5 CONCLUSIONS

Due to the random errors of georeferenced measurements, the polygon angles of the building object are not congruent with both actual and design angles of the building object. However, it is possible to make an attempt to bring about congruency with these angles. One of the possibilities, proposed in this paper, is the adjustment of the polygon using the least squares method robust to polygon angles, much outlying from their design values. The conducted experiment on real data has confirmed the efficacy of this method. All of the six used methods of robust estimation (Hampel, Krarup, Kraus, Yang, Huber and Huber modified) have detected the same outlying angles. The adjusted polygon of the building object has angles at fourteen points equal to the design angles and detected angles at two points that are much outlying from the design ones, with determined values of deviations. In accordance with the expectation, the positions of all sixteen polygon points slightly moved within two times of values of their standard deviations. The reason for the much outlying detected angles at two points (11 and 16) can result from the measurement error of their coordinates or errors of the object construction. The determination of a reason would require to perform the control measurement of coordinates of these two points. The proposed method can be used to correct the shape of directly measured building objects as well as to adjust the shape of buildings that are a part of geospatial databases, for which the standard deviations of the polygon points are known.

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