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DEFORMACIJSKA ANALIZA PO MODIFICIRANI METODI GREDOD

DEFORMATION ANALYSIS: THE MODIFIED GREDOD METHOD

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IZVLEČEK

V prispevku je predstavljena modificirana splošna robustna ocena deformacij iz razlike opazovanj (GREDOD), ki temelji na uporabi genetskega algoritma (GA) in splošnega algoritma optimizacije roja delcev (GPSO) za reševanje optimizacijskega problema te metode, pri čemer gre za problem določitve optimalnega datuma vektorja premika. Postopek deformacijske analize s to modifikacijo metode GREDOD je prikazan na primeru dvodimenzionalne geodetske mreže, predstavljene v številnih raziskavah, v katerih so bila simulirana vsa opazovanja in premiki. Z uporabo obeh algoritmov GA in GPSO smo dobili skoraj enake rezultate deformacijske analize kot z metodami Hannover, Karlsruhe, Delft, Fredericton, München, Caspary in klasičnimi robustnimi metodami, le rešitve datuma vektorja premika so popolnoma različne.

ABSTRACT

In this paper, a modified Generalised Robust Estimation of Deformation from Observation Differences (GREDOD) method is presented, based on the application of genetic algorithm (GA) and generalised particle swarm optimisation (GPSO) algorithm in solving the optimisation problem of this method, which is, in essence, a problem of determining the optimal datum of the displacement vector. The procedure of deformation analysis using this modification of the GREDOD method is demonstrated in the example of the two-dimensional geodetic network presented in numerous research and in which all observations and displacements were simulated. Using both algorithms, GA and GPSO, almost identical results of deformation analysis were obtained, except datum solutions of the displacement vector, which are completely different. These results differ only slightly from the results obtained using the methods of Hannover, Karlsruhe, Delft, Fredericton, München, Caspary, and the classical robust method.

KLJUČNE BESEDE

robustna deformacijska analiza, genetski algoritem, splošni algoritem optimizacije roja delcev

KEY WORDS

Robust deformation analysis, Genetic algorithm, Generalised particle swarm optimisation

1 INTRODUCTION

The surface of the Earth's crust and the engineering facilities built on it are subject to displacements and deformations occurring as a consequence of the influence of various factors, such as tectonic and seismic activities, landslides, earthquakes, changes in temperature and groundwater levels, material fatigue, errors in the design and execution of construction works, etc. Today, there are many different measurement methods for the detection and identification of displacements and deformations, which can be divided into geodetic and non-geodetic (Marković et al., 2019). Geodetic methods are based on examining the temporal evolution of geodetic networks, which are realised by physically stabilised points in the surface of the Earth's crust. Geodetic networks that are established for the purpose of monitoring displacements and deformations can be divided into absolute and relative networks (Chrzanowski, Chen, and Secord, 1986; Caspary, 2000; Setan and Singh, 2001). Absolute geodetic networks consist of points that represent the object in a geometric sense and potential reference points (PRPs) stabilised outside the zone of expected deformations. On the other hand, relative geodetic networks consist only of points on the object or area that is the subject of monitoring. The relative networks are usually used in deformation analysis of the surface of the Earth's crust, whereas absolute networks are usually used in deformation analysis of engineering facilities, such as dams, bridges, towers, tunnels, etc. It is important to note that these geodetic networks must meet specific reliability, precision, and sensitivity requirements. Information about displacements and deformations in geodetic networks are obtained based on two or more measurement epochs realised in different periods or based on permanent observations when it comes to automated real-time monitoring systems.

Deformation analysis of geodetic networks is a very important segment of geodetic monitoring of displacements and deformations of engineering facilities and the Earth's crust surface. Methods of deformation analysis of geodetic networks have been the subject of intensive research by scientists in the last fifty years. Among the abundance of methods of deformation analysis, the following methods are particularly interesting: Hannover (Pelzer, 1971), Delft (Heck et al., 1982), Karlsruhe (Heck, 1983), Fredericton (Chrzanowski, Chen and Secord, 1982; Chen, 1983), München (Welsch, 1982) and Caspary's method (Caspary, 1987). These methods are represented in numerous research (Ambrožič, 2001, 2004; Setan and Singh, 2001; Mozetič, Kogoj and Ambrožič, 2006; Hekimoglu, Erdogan and Butterworth, 2010; Marjetič, Zemljak and Ambrožič, 2012; Vrečko and Ambrožič, 2013; Sušić et al., 2015, 2017; Soldo and Ambrožič, 2018; Hamza, Stopar and Ambrožič, 2020) and are very well established in the literature and practice.

Neitzel (2004) proposed the maximum subsample (MSS) method which uses combinatorial search to find the largest congruent group of points in the time interval between two measurement epochs. However, the number of possible combinations progressively increases with the increasing number of points in the geodetic network, especially in large geodetic networks with only several stable (undisplaced) points. The number of combinations is reduced by applying two strategies: MSS using distance differences and MSS using distance ratio. Both strategies quickly and easily find combinations of points that are potentially the largest congruent group of points based on an appropriate matrix of topological relations. It is important to point out that MSS using distance differences is applicable only in cases when the geodetic network scale is the same in two measurement epochs, while MSS using distance ratio is applicable and when the geodetic network scale is not the same in both measurement epochs. Ebeling (2014) developed a new strategy MSS using angles, which is based on the analysis of angular differences between epochs and is an alternative to MSS using distance ratio. In recent years, Velsink (2015), Lehmann and Lösler (2017) and Nowel (2020) proposed combinatorial methods that overcome the weakness of previously listed methods, such as so-called displacement smearing.

Recently, robust methods of deformation analysis have become increasingly popular and represented in scientific research. The best-known robust method, the iterative weighted similarity transformation (IWST), was developed in 1983 at the University of New Brunswick, Canada (Chen, 1983). An alternative method called generalised robust estimation of deformation from observation differences (GREDOD) was proposed by Polish scientists Nowel and Kamiński (2014) and Nowel (2015). The main difference between these two methods is in the methodology of robust estimation of the displacement vector. IWST method calculates the estimated displacement vector based on the difference between adjusted coordinates of geodetic network points in two measurement epochs. On the other hand, in the GREDOD method, the displacement vector is determined based on the differences of unadjusted observations from two measurement epochs. Numerous modifications of the IWST method based on the introduction of different optimisation conditions of robust estimation are also present in the literature (Caspary and Borutta, 1987; Nowel, 2015; Ambrožič et al., 2019). For both IWST and GREDOD methods, in the procedure of robust estimation of the displacement vector, which is, in essence, the optimisation problem, the iterative reweighted least-squares (IRLS) method is traditionally applied. This method starts from an initial solution obtained using the least-squares method and iteratively improves this solution during the optimisation process. However, if the initial solution is not in the vicinity of the global one, the IRLS method is only capable to determine the local optimum (Baselga, 2007). In order to overcome this flaw, Batilović et al. (2021, 2022) proposed modifications of the IWST and GREDOD methods based on the application of two evolutionary optimisation algorithms, genetic algorithm (GA) and generalised particle swarm optimisation (GPSO) algorithm, in the procedure of robust estimation of the displacement vector instead of the IRLS method. Experimental analysis of these modifications was conducted using the mean success rate (MSR) based on Monte Carlo simulations. The obtained results showed that the efficacy of the IWST and GREDOD methods was significantly improved by applying the GA and GPSO algorithms.

In this paper, a modified GREDOD method is presented, in which the optimisation condition of robust estimation of the displacement vector is defined by Huber's objective function. The procedure of application of GA and GPSO algorithms in the process of robust estimation of the displacement vector is described in detail. This modification of the GREDOD method was applied to one set of simulated observations (zero and control measurement epoch) in the two-dimensional geodetic network, which is represented in numerous research. The obtained results are compared with the results of Hannover, Karlsruhe, Delft, München, Fredericton, Caspary, and the classical robust method (Huber).

2 MODIFIED GREDOD METHOD

In general, the GREDOD method consists of two phases. In the first phase, robust estimation of the displacement vector from the difference in unadjusted observations is performed. The second phase involves testing the stability of network points, i.e., examining whether the estimated single-point displacements result from actual displacements or only measurement errors. The GREDOD method eliminates the influence of systematic errors, which can burden the results of observations in certain measurement epochs, on the results of deformation analysis.

2.1 Deformation model and optimisation problem

The GREDOD method is based on the following deformation model:

$$\Delta\mathbf{l} + \mathbf{v}_\Delta = \mathbf{A}\mathbf{d}, \tag{1}$$

$$\Delta\mathbf{l} \sim N(\mathbf{A}\mathbf{d}, \mathbf{C}_\Delta), \tag{2}$$

where $\Delta\mathbf{l} = \mathbf{l}_2 - \mathbf{l}_1$ is the vector of observation differences, $\mathbf{v}_\Delta = \mathbf{v}_2 - \mathbf{v}_1$ is the vector of the residuals of observation differences, $\mathbf{A} = \mathbf{A}_1 = \mathbf{A}_2$ is the design matrix, $\mathbf{d} = \mathbf{x}_2 - \mathbf{x}_1$ is the displacement vector, $\mathbf{C}_\Delta = \sigma_0^2\mathbf{P}_\Delta^{-1}$ is the covariance matrix of observation differences, $\mathbf{P}_\Delta = (\mathbf{P}_1^{-1} + \mathbf{P}_2^{-1})^{-1}$ is the weight matrix of observation differences and σ_0^2 is the *a priori* variance factor (Nowel and Kamiński, 2014). In this model, the displacement vector is a vector of unknown parameters, and the observation vector is a vector of the observation differences from two measurement epochs.

The optimisation problem of the GREDOD method defines the previously formulated deformation model and the following objective functions:

$$\mathbf{v}_\Delta^T\mathbf{P}_\Delta\mathbf{v}_\Delta = min, \tag{3}$$

$$\rho(\mathbf{d}) = min. \tag{4}$$

The function (4) can be any objective function from the robust M estimation class. This optimisation problem, which is actually the problem of determining the optimal datum of the displacement vector, was solved using the Lagrangian multiplier method and the theory of generalised inverses of matrices (Nowel, 2015). The solution of the optimisation problem is the following estimator of displacement vector (Batilović et al., 2022):

$$\hat{\mathbf{d}} = \mathbf{R}\Delta\mathbf{l}, \tag{5}$$

where $\mathbf{R} = \mathbf{W}^{-1}\mathbf{N}_\Delta(\mathbf{N}_\Delta\mathbf{W}^{-1}\mathbf{N}_\Delta + \mathbf{B}\mathbf{B}^T)^{-1}\mathbf{N}_\Delta\mathbf{W}^{-1}\mathbf{N}_\Delta(\mathbf{N}_\Delta\mathbf{W}^{-1}\mathbf{N}_\Delta + \mathbf{B}\mathbf{B}^T)^{-1}\mathbf{A}^T\mathbf{P}_\Delta$, $\mathbf{N}_\Delta = \mathbf{A}^T\mathbf{P}_\Delta\mathbf{A}$, $\mathbf{W}$ is a diagonal weight matrix of the displacement vector, $\mathbf{B}$ is the matrix of rank *de* that is formed in the same way as in the classical approach of defining the datum of geodetic networks, as explained in (Caspary, 2000), and *de* is the defect datum of the geodetic network. The weight matrix $\mathbf{W}$ has the following form:

$$\mathbf{W} = diag(\ldots, w_{PRP,i}, \ldots, w_{O,i}, \ldots). \tag{6}$$

Elements $w_{O,i}$ that refer to the object points must have very small values close to zero, e.g., $w_{O,i} = 10^{-4}$. On the other hand, $w_{PRP,i} = w(\hat{d}_{PRP,i})$ is the appropriate weight function from the robust M estimation class (Nowel, 2015). It is obvious that equation system (5) cannot be solved directly because displacements in $w(\hat{d}_{PRP,i})$ are unknown.

2.2 Robust estimation of the displacement vector using GA and GPSO algorithms

The optimisation problem of the GREDOD method, more precisely equation system (5), can be solved using GA and GPSO algorithms. For the purpose of applying these algorithms, it is necessary to define variables, the objective function, as well as the appropriate constraints for variables. The main instance in these two algorithms is the so-called individual (in GA) or particle (in GPSO), which represents a potential solution, i.e., a set of variables defined by an optimisation problem. Since the weights of PRPs represent variables, the individual (particle) is defined as a weight vector of PRPs (Batilović et al., 2021, 2022):

$$\mathbf{y} = \left[w_{PRP,1}, w_{PRP,2}, \ldots, w_{PRP,n} \right]. \tag{7}$$

In this paper, the Huber function is used as an objective function (4):

$$\rho(\mathbf{d}) = \sum \rho(d_i) = min,$$
$$\rho(d_i) = \begin{cases} \dfrac{d_i^2}{2} & \text{for } |d_i| \le c\hat{\sigma}_{d_i} \\ c\hat{\sigma}_{d_i} |d_i| - \dfrac{(c\hat{\sigma}_{d_i})^2}{2} & \text{for } |d_i| > c\hat{\sigma}_{d_i} \end{cases}, \tag{8}$$

where d_i are components of the displacement vector, $c\hat{\sigma}_{d_i}$ is the tuning constant, c is a suitable factor (e.g., $c = 1.345$), and $\hat{\sigma}_{d_i}$ is the least-square estimator of the standard deviation (Caspary and Borutta, 1987). In this objective function, displacements d_i are divided into two groups, namely: displacements whose absolute values are less than or equal to the constant $c\hat{\sigma}_{d_i}$ and displacements whose absolute values are greater than constant $c\hat{\sigma}_{d_i}$. The Huber weight function has the following form:

$$w_{PRP,i} = \begin{cases} 1 & \text{for } \left|\hat{d}_i\right| \le c\hat{\sigma}_{d_i} \\ \dfrac{c\hat{\sigma}_{d_i}}{\left|\hat{d}_i\right|} & \text{for } \left|\hat{d}_i\right| > c\hat{\sigma}_{d_i} \end{cases}. \tag{9}$$

The codomain of this weight function is [0, 1]. However, the GA and GPSO algorithms do not use equation (9) to calculate the weights of PRPs but start from a randomly chosen set of individuals (particles) (7) and improve them iteratively during the optimisation process. For this reason, the following constraint for the weights of PRPs is defined

$$10^{-4} \le w_{PRP,i} \le 1, \tag{10}$$

which is integrated into the objective function (8) by the penalty functions method, in the following way

$$\rho(\mathbf{d}) = \sum \rho(d_i) + \sum_{i=1}^{n} \beta g_i = min,$$
$$\rho(d_i) = \begin{cases} \dfrac{d_i^2}{2} & \text{for } |d_i| \le c\hat{\sigma}_{d_i} \\ c\hat{\sigma}_{d_i} |d_i| - \dfrac{(c\hat{\sigma}_{d_i})^2}{2} & \text{for } |d_i| > c\hat{\sigma}_{d_i} \end{cases}, \tag{11}$$

where g_i is the corresponding penalty for each variable and β is the weight coefficient of the penalty. In this method, for every constraint violation the corresponding penalty g_i is formed

$$g_i = \begin{cases} \left| w_{PRP,i} - w_{max} \right|, w_{PRP,i} > w_{max} \\ 0, w_{min} \leq w_{PRP,i} \leq w_{max} \\ \left| w_{min} - w_{PRP,i} \right|, w_{PRP,i} < w_{min} \end{cases}, \tag{12}$$

where $w_{min} = 10^{-4}$ and $w_{max} = 1$ (Kramer, 2010; Jordehi, 2015). If the obtained solution satisfies constraint (10), the objective functions (8) and (11) are identical, because the corresponding penalties g_i are equal to zero. On the other hand, if a solution that exceeds the constraint (10) appears during the optimisation process, the corresponding penalty g_i increases the value of the objective function (11), which results in the elimination of that solution. Hence, it is evident that the final optimal solution will be within the defined bounds.

Algorithm 1: Pseudocode of genetic algorithm.

```
begin
    k ← 0
    generate initial population (randomly created set of individuals)
    create weight matrix W (6) for each individual y
    calculate displacement vector d̂ (5) for each individual
    calculate value of objective function (11) for each individual y
    while not stopping criterion do
        k ← k + 1
        select two individuals from the old generation for crossover
        bool test = probability test for crossover
        if test then
            perform crossover of two individuals to create two new individuals
        else
            new individuals = old individuals
        end
        bool test = probability test for mutation
        if test then
            perform mutation of new individuals
        end
        insert new individuals into population replacing old ones
        create weight matrix W (6) for each individual y
        ulate displacement vector d̂ (5) for each individual y
        calculate value of objective function (11) for each individual y
    end
end
```

The genetic algorithm starts from a randomly selected set of individuals called the initial population. For each individual (7) in the population, a corresponding weight matrix is formed using equation (6), after which the estimated displacement vector (5) and the value of the objective function (11) are determined. The value of the objective function (11) is called fitness and represents the individual's quality. The initial population changes iteratively using evolution mechanisms of selection (for selecting the best individuals for reproduction), crossover (combining the variable values of selected individuals) and mutation (slight changes in the variables of individuals selected with a very small probability) until one of the stopping criteria (total iteration number, tolerance, accuracy, or calculation time) is fulfilled, as explained in Algorithm 1. Iterations k are terminologically defined as generations, where each new generation brings improvements in the population, to eventually converge toward the global optimal solution. The individual with the best value of the objective function (11) from the last generation represents the optimal solution. The estimated displacement vector $\hat{\mathbf{d}}$ and corresponding cofactor matrix $\mathbf{Q}_{\hat{\mathbf{d}}} = \mathbf{R}\mathbf{P}_{\Delta}^{-1}\mathbf{R}^{T}$ are determined using the weights values $w_{PRP,i}$ of this individual. The working principle of the GA is described in more detail in the literature (Goldberg, 1989; Mitchell, 1999).

The initial step in the GPSO algorithm is to create an initial swarm which consist of a set of randomly selected particles. Each particle (7) is determined by a set of variables' values, which is interpreted as the "position" of the particle in the search space. During the optimisation process, every particle memorises its best position ($\mathbf{p}$), where it achieved the best value of the objective function (11), as well as the global best position achieved by all particles in the swarm ($\mathbf{g}$). In order to find the optimal solution, the particles are iteratively "repositioned" in the search space using the following expression

$$\mathbf{y}^{(k+1)} = (1 - 2\zeta\rho + \rho^2)(c \cdot \mathbf{p}^{(k)} + (1 - c)\cdot \mathbf{g}^{(k)}) + 2\zeta\rho\mathbf{y}^{(k)} - \rho^2\mathbf{y}^{(k+1)}, \tag{13}$$

where ρ, ζ and c parameters of GPSO algorithms, and k is the iteration number. The iterative procedure is conducted until one of the stopping criteria is fulfilled, as explained in Algorithm 2. After the adopted stopping criterion is fulfilled, the particle providing the best value of the objective function (11) is adopted as the optimal solution. Based on this particle, i.e., the values of its weights $w_{PRP,i}$, the estimated displacement vector $\hat{\mathbf{d}}$ and the cofactor displacement matrix $\mathbf{Q}_{\hat{\mathbf{d}}} = \mathbf{R}\mathbf{P}_{\Delta}^{-1}\mathbf{R}^{T}$ are determined. Detailed information on the GPSO algorithm can be found in (Kanović, Rapaić and Jeličić, 2011).

Algorithm 2: Pseudocode of GPSO algorithm.

```
begin
    k ← 0
    generate initial swarm (randomly created set of particles)
    while not stopping criterion do
        k ← k + 1
        for (i = 1 to n) do
            calculate new position of particle y_i using equation (13)
            create weight matrix W (6) for particle y_i
            calculate displacement vector d̂ (5) for particle y_i
            calculate value of objective function (11) for particle y_i
            update the personal best position p_i
        end
        update the global best position g
    end
end
```

2.3 Significance test of single-point displacements

The statistical significance test of single-point displacements is based on the null and alternative hypotheses

$$H_0 : E(\hat{\mathbf{d}}_i) = 0 \quad \text{against} \quad H_a : E(\hat{\mathbf{d}}_i) \neq 0, \tag{14}$$

where $\hat{\mathbf{d}}_i$ is the estimated displacement vector of the *i*th point, and E is the mathematical expectation operator. If the null hypothesis H_0 is not rejected, the displacement of the *i*th point is not statistically significant, and this point can be regarded as stable. On the other hand, if the null hypothesis is rejected, the *i*th point is regarded to be unstable, which means that the displacement of this point is statistically significant. The null hypothesis can be accepted if the following condition is fulfilled

$$\hat{\mathbf{d}}_i \in E_\alpha = f(F_{1-\alpha_0,u_i,r}, \hat{\sigma}_0^2, \mathbf{Q}_{\hat{\mathbf{d}}_i}), \tag{15}$$

where E_α is the confidence interval (region), $F_{1-\alpha_0,u_i,r}$ is the quantile of F-distribution, $\alpha_0 = 1-(1-\alpha)^{1/m} \cong \alpha/m$ is the local significance level (α is the global significance level), $u_i = rank(\mathbf{Q}_{\hat{\mathbf{d}}_i})$, $r = n-u+de$ is the number of degrees of freedom, n is the number of observations, u is the number of unknown parameters, de is the defect datum of the geodetic network, $\hat{\sigma}_0^2 = (\hat{\mathbf{v}}_\Delta^T \mathbf{P}_\Delta \hat{\mathbf{v}}_\Delta)/r$ is the *a posteriori* variance factor and $\mathbf{Q}_{\hat{\mathbf{d}}_i}$ is the displacement cofactor matrix of the *i*th point (Chen, 1983; Nowel and Kamiński, 2014). In this approach, the evaluation of statistical significance consists of checking graphically whether the estimated displacement vector $\hat{\mathbf{d}}_i$ does not exceed the confidence interval (1D network) or the confidence ellipse (2D network) (Kamiński and Nowel, 2013) or the confidence ellipsoid (3D network) (Cederholm, 2003).

The parameters of the confidence ellipse are the semi-major axis A_i, the semi-minor axis B_i, and the twist angle of the ellipse θ_i, defined as:

$$A_i = \hat{\sigma}_0 \sqrt{2\lambda_{1,i} F_{1-\alpha_0,u_i,r}}, \tag{16}$$

$$B_i = \hat{\sigma}_0 \sqrt{2\lambda_{2,i} F_{1-\alpha_0,u_i,r}}, \tag{17}$$

$$\theta_i = \frac{1}{2} atan\left(\frac{2Q_{\hat{d}_{x,i}\hat{d}_{y,i}}}{Q_{\hat{d}_{x,i}} - Q_{\hat{d}_{y,i}}} \right), \tag{18}$$

with

$$\lambda_{1,i} = (Q_{\hat{d}_{x,i}} + Q_{\hat{d}_{y,i}} + \Delta_i)/2,$$

$$\lambda_{2,i} = (Q_{\hat{d}_{x,i}} + Q_{\hat{d}_{y,i}} - \Delta_i)/2,$$

$$\Delta_i = \sqrt{(Q_{\hat{d}_{x,i}} - Q_{\hat{d}_{y,i}})^2 + 4Q_{\hat{d}_{x,i}\hat{d}_{y,i}}{}^2},$$

where $Q_{\hat{d}_{x,i}}, Q_{\hat{d}_{y,i}}$ and $Q_{\hat{d}_{x,i}\hat{d}_{y,i}}$ the elements of displacement cofactor matrix $\mathbf{Q}_{\hat{\mathbf{d}}_i}$.

3 NUMERICAL EXAMPLE

In this paper, the experimental research was conducted on the example of a two-dimensional geodetic network, which has been represented in numerous research in the last two decades (Ambrožič, 2001, 2004; Marjetič, Zemljak and Ambrožič, 2012; Vrečko and Ambrožič, 2013; Soldo and Ambrožič, 2018; Ambrožič et al., 2019; Hamza, Stopar and Ambrožič, 2020). Therefore, the obtained results are directly comparable with the results presented in the cited publications. This geodetic network consists of seven potential reference points. Two measurement epochs were simulated in the geodetic network (Figure 1), and each of them consists of 24 horizontal directions and 24 horizontal distances. Observations are simulated with random measurements errors that follow a normal distribution with a mean value of zero and standard deviations $\sigma_\alpha = 1''$ and $\sigma_d = 5$ mm for horizontal directions and distances, respectively. The dataset contains $n = 48$ observations and $u = 21$ unknown parameters, more precisely 14 unknown coordinates and 7 unknown orientations. The datum defect of the geodetic network de, the number of degrees of freedom r and mean redundancy number $\bar{r}$ are 3, 30, and 0.625, respectively. Approximate coordinates of geodetic network points and results of simulated observations in two epochs are available in Ambrožič (2001). Sketch of geodetic network with simulated displacement vectors and error ellipses for the confidence level of 95% is shown in Figure 1.

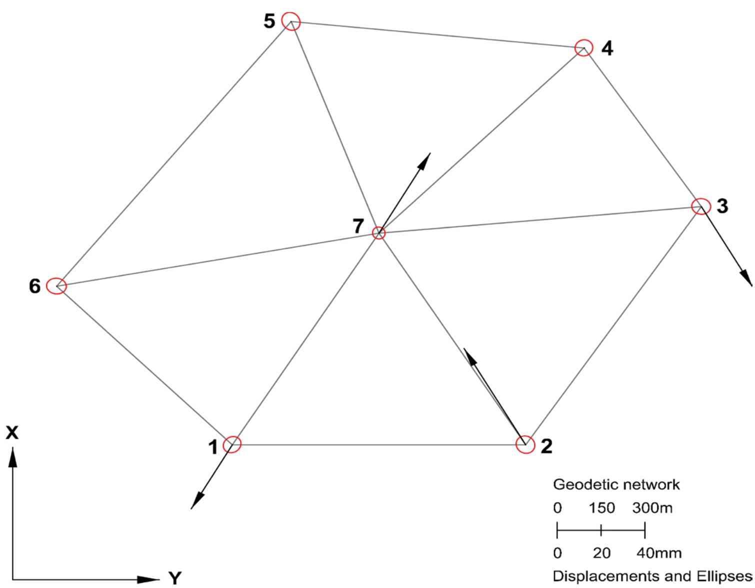

Figure 1: Geodetic network with error ellipses and simulated displacements.

Deformation analysis of geodetic network was performed using GREDOD method, whereby the GA and GPSO algorithms were applied in the procedure of robust estimation of the displacement vector. The optimisation condition of the robust estimation is formulated by Huber's objective function, and the value of 1.345 is adopted for suitable factor c. In order to apply GA and GPSO algorithms in the procedure of robust estimation of the displacement vector, a constraint (10) for the weight values $w_{PRP,i}$ was defined. This constraint was integrated into the objective function (11) by the penalty function method, where the value 10^6 was adopted for the weight coefficient of the penalty β. Parameters of genetic algorithm were adopted based on recommendations in the literature (Goldberg, 1989; Mitchell, 1999). Stochastic uniform selection with linear ranging, uniform crossover scheme, and Gaussian mutation have been applied. The change of generations is performed by applying an elitist strategy, which implies the direct transfer of 5% of the best individuals to the next generation without the use of genetic operators (selection, crossover, and mutation). The GPSO parameter ζ takes values from the range [−0.9, 0.2] using a uniform distribution, while the GPSO parameters ρ and c decrease linearly within the ranges [0.95, 0.60] and [0.8, 0.2], respectively, during the search process, as proposed in (Kanović, Rapaić and Jeličić, 2011). For the number of individuals in the GA, i.e., the number of particles in the GPSO algorithm, the adopted value is 1000. The stopping criterion is defined by the tolerance and the maximum number of generations (iterations), whereby the values 10^{-6} and 150 were adopted for these parameters. Values of the number of individuals (particles), tolerance, and a maximum number of generations (iterations) are determined based on experiments, observing the convergence of the optimisation process.

It is well known that when applying GA and GPSO algorithms it is not possible to repeat all the steps in the optimisation process for an identical problem and the same initial conditions because the solution space search is performed in a controlled random manner. Therefore, the procedure of robust estimation of displacement vectors using GA and GPSO algorithms was repeated 100 times, to increase the reliability and representativeness of the obtained results. Table 1 shows the minima, maxima, means, and medians of the objective function (11) obtained in the case of application GA and GPSO algorithms.

Table 1: Characteristic values of the objective function.

Algorithms	Minimum	Maximum	Mean	Median
GA	891.6651081686	891.6651085811	891.6651081821	891.6651081780
GPSO	891.6651081679	891.6651081822	891.6651081749	891.6651081752

Based on these values, one can conclude that the choice of the parameters for both algorithms is appropriate, since all values differ very slightly, i.e., in all cases the solution is near-global optimum. The solutions that have the best, i.e., the minimum, values of the objective function (11) were adopted for the final solutions. Figure 2 depicts a flow of the optimisation process of the GA and GPSO algorithms. Generations (iterations) are shown on the abscissa, and the values of the objective function in the logarithmic scale are shown on the ordinate. It can be observed that the genetic algorithm converges faster towards the optimal solution.

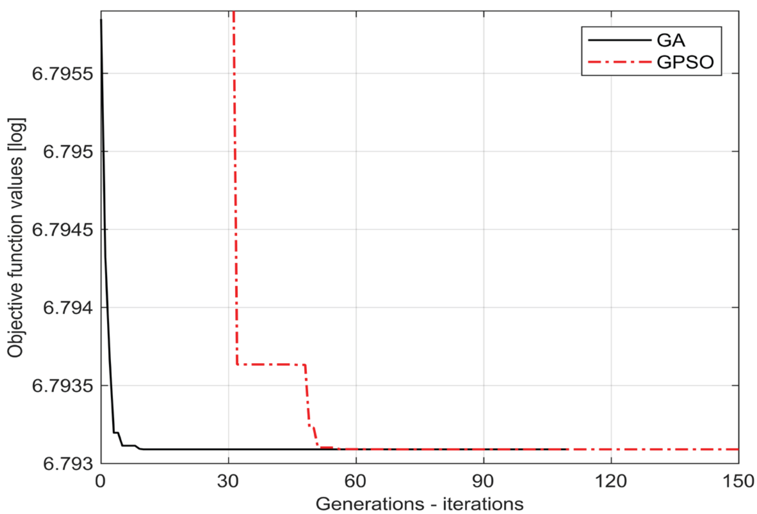

Figure 2: Flow of the optimisation process of the GA and GPSO algorithms.

The second phase in the deformation analysis procedure is the statistical significance test of single-point displacements. For the global significance level α the value 0.05 was adopted. The local significance level α_0 was calculated using the Bonferroni equation and it is 0.0073. It should be emphasised that the analysis stability of points was performed graphically. This approach consists in checking graphically whether the estimated displacement vector of *i*th point does not exceed the confidence ellipse of *i*th point.

Simulated displacement vectors and the results of the deformation analysis are presented in Table 2. It is obvious that by applying these algorithms, completely different optimal values of weights $w_{PRP,i}$ were obtained, i.e., the datum solutions for the displacement vector of geodetic network points are different. However, despite the different datum solutions, identical values of the components of the estimated displacement vector were obtained for both algorithms. Points 1, 2, 3, and 7 were identified as unstable while points 4, 5, and 6 were identified as stable, as can be seen in Figure 3. Since the values of the confidence ellipses parameters differ very slightly in the case of GA and GPSO algorithms, graphical representation of the confidence ellipses obtained by the GPSO algorithm is omitted.

Table 2: Simulated displacements and results of the deformation analysis.

Points	**Simulated**	**GA**				**GPSO**			
	d_{Y_i} [mm]	$\hat{d}_{Y_i}$ [mm]	A_i [mm]	$w_{PRP,i}$	Stable	$\hat{d}_{Y_i}$ [mm]	A_i [mm]	$w_{PRP,i}$	Stable
	d_{X_i} [mm]	$\hat{d}_{X_i}$ [mm]	B_i [mm]			$\hat{d}_{X_i}$ [mm]	B_i [mm]		
	d_i [mm]	$\hat{d}_i$ [mm]	θ_i [°]			$\hat{d}_i$ [mm]	θ_i [°]		
	-20.00	-14.61	10.31	0.257		-14.61	10.42	0.283	
1	-34.60	-37.91	8.09	0.882	**No**	-37.91	8.04	0.974	**No**
	40.00	40.63	79.95			40.63	76.77		
	-30.00	-33.39	10.41	0.638		-33.39	10.00	0.599	
2	52.00	52.77	8.55	0.567	**No**	52.77	8.30	0.874	**No**
	60.00	62.44	139.51			62.44	131.42		
	25.00	22.95	10.87	0.240		22.95	10.15	0.331	
3	-43.30	-38.08	9.41	0.382	**No**	-38.08	8.62	0.625	**No**
	50.00	44.47	127.93			44.47	111.39		
	0.00	0.10	9.43	0.822		0.10	9.54	0.542	
4	0.00	4.98	8.32	0.586	Yes	4.98	9.17	0.592	Yes
	0.00	4.98	37.15			4.98	30.19		
	0.00	-1.80	11.89	0.979		-1.80	10.88	0.771	
5	0.00	-1.86	7.75	0.070	Yes	-1.87	8.18	0.531	Yes
	0.00	2.60	176.00			2.60	167.72		
	0.00	2.72	9.91	0.605		2.72	10.33	0.412	
6	0.00	-1.20	7.62	0.893	Yes	-1.20	8.09	0.789	Yes
	0.00	2.98	87.69			2.98	96.40		
	25.00	25.90	7.64	0.756		25.90	7.57	0.647	
7	43.30	44.40	6.22	0.368	**No**	44.40	6.17	0.306	**No**
	50.00	51.41	170.08			51.41	175.40		

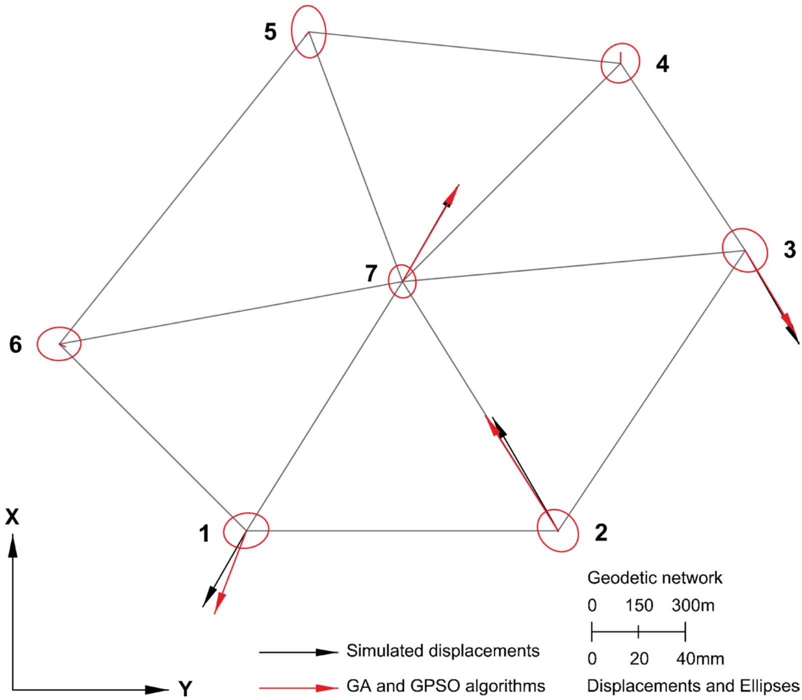

Figure 3: Geodetic network with simulated displacements, estimated displacements, and confidence ellipses.

The obtained results are compared with results from the methods of Hannover, Karlsruhe, Delft, Fredericton, München, Caspary, and the classical robust method, where the optimisation condition is formulated by Huber's objective function. Information about stability, simulated and estimated displacements of points are shown in Table 3. It is evident that the values of the estimated displacement vectors in the classical robust and GREDOD methods differ slightly from the estimated displacement vectors in other methods. However, the values of the estimated displacements are very close to the simulated displacements in all deformation analysis methods. All points at which displacements were simulated, i.e., points 1, 2, 3, and 7, were identified as displaced (unstable) in all methods. Points 4, 5, and 6 were identified as undisplaced (stable) in all methods except in the case of the classical robust method where point 4 was identified as displaced (unstable).

Table 3: Simulated displacements and results from deformation analysis by Hannover, Karlsruhe, Delft, Fredericton, München, Caspary, classical robust (Huber), and GREDOD methods.

Points	Methods	Simulated	Hanover	Karlsruhe	Delft	Fredericton	München	Caspary	Classical robust method - Huber	GREDOD -Huber
1	ˆ [mm]	-20.00	-19.60	-19.70	-19.40	-19.60	-19.50	-19.20	-14.50	-14.61
	$\hat{d}_X$ [mm]	-34.60	-38.00	-38.00	-37.50	-38.00	-37.60	-37.90	-37.70	-37.91
	$\hat{d}$ [mm]	40.00	42.80	42.80	42.20	42.80	42.40	42.50	40.40	40.63
	v [°]	210.00	207.00	207.00	207.00	207.00	207.00	207.00	201.04	201.08
	Displaced	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**
2	$\hat{d}_Y$ [mm]	-30.00	-38.70	-38.80	-38.10	-38.70	-38.20	-38.40	-33.20	-33.39
	$\hat{d}_X$ [mm]	52.00	49.00	49.00	49.50	49.00	49.50	49.40	53.00	52.77
	$\hat{d}$ [mm]	60.00	62.40	62.50	62.50	62.50	62.50	62.50	62.50	62.44
	v [°]	330.00	322.00	322.00	322.00	322.00	322.00	322.00	327.94	327.68
	Displaced	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**
3	$\hat{d}_Y$ [mm]	25.00	20.60	20.60	21.40	20.60	21.40	20.80	23.00	22.95
	$\hat{d}_X$ [mm]	-43.30	-44.30	-44.40	-43.50	-44.30	-43.60	-43.90	-37.70	-38.08
	$\hat{d}$ [mm]	50.00	48.90	48.90	48.50	48.90	48.60	48.60	44.20	44.47
	v [°]	150.00	155.00	155.00	154.00	155.00	154.00	154.00	148.61	148.92
	Displaced	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**
4	$\hat{d}_Y$ [mm]	0.00	-4.00	–	0.70	–	0.70	–	0.10	0.10
	$\hat{d}_X$ [mm]	0.00	5.10	–	1.00	–	1.00	–	5.20	4.98
	$\hat{d}$ [mm]	0.00	6.50	–	1.20	–	1.20	–	5.20	4.98
	v [°]	–	322.00	–	35.00	–	35.00	–	1.10	1.15
	Displaced	No	No	No	No	No	No	No	**Yes**	No
5	$\hat{d}_Y$ [mm]	0.00	-6.40	–	-0.80	–	-0.80	–	-1.80	-1.80
	$\hat{d}_X$ [mm]	0.00	-7.10	–	-2.30	–	-2.20	–	-1.80	-1.86
	$\hat{d}$ [mm]	0.00	10.00	–	2.40	–	2.30	–	2.50	2.60
	v [°]	–	222.00	–	199.00	–	200.00	–	225.00	224.06
	Displaced	No	No	No	No	No	No	No	No	No
6	$\hat{d}_Y$ [mm]	0.00	3.30	–	0.00	–	0.00	–	2.70	2.72
	$\hat{d}_X$ [mm]	0.00	-10.60	–	1.30	–	1.40	–	-1.10	-1.20
	$\hat{d}$ [mm]	0.00	11.10	–	1.30	–	1.40	–	2.90	2.98
	v [°]	–	163.00	–	0.00	–	0.00	–	112.17	113.81
	Displaced	No	No	No	No	No	No	No	No	No
7	$\hat{d}_Y$ [mm]	25.00	23.60	23.60	24.00	23.60	24.00	23.90	25.90	25.90
	$\hat{d}_X$ [mm]	43.30	42.90	42.90	42.90	42.90	42.90	43.10	44.60	44.40
	$\hat{d}$ [mm]	50.00	49.00	49.00	49.20	48.90	49.20	49.30	51.60	51.41
	v [°]	30.00	29.00	29.00	29.00	29.00	29.00	29.00	30.14	30.26
	Displaced	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**

4 CONCLUSION

This paper has presented a modified GREDOD method based on application GA and GPSO algorithms in the procedure of robust estimation of the displacement vector, i.e., determination of the optimal datum solution of the displacement vector. The application of these two evolutionary optimisation algorithms has been proposed to overcome the main disadvantage of the IRLS method, the inability to determine the global optimal datum solution of displacement vector in some specific cases when there are unstable (displaced) points in the set of datum points. In order to apply the GA and GPSO algorithms, the individual (particle) (7), which represent the main instance in these algorithms, was defined as the weights vector of PRPs. Since the optimisation condition of robust estimation is defined by the Huber objective function (8), constraint (10) for the weights of PRPs was defined based on the appropriate weight function (9). This constraint, which actually defines the feasible search region, is integrated into the objective function (11) by the penalty functions method. In this method, for every constraint violation, the appropriate penalty (12) is formed based on the distance of the obtained solution from the feasible search region. In that manner, solutions that exceed the defined constraint are eliminated.

The procedure of deformation analysis using the modified GREDOD method is demonstrated on one set of simulated observations that consists of two measurement epochs in the two-dimensional geodetic network. In the case of applying both GA and GPSO algorithms identical single-point displacements were obtained, which are very close to the simulated displacements. All displaced points were identified as unstable, while all undisplaced points were identified as stable using both algorithms. However, it should be emphasised that in the case of application of the GA and GPSO algorithms different datum solutions of the displacement vector were obtained. The obtained results differ slightly from the results obtained by other methods of deformation analysis presented in previously cited articles, which confirms the efficacy of the modified GREDOD method.

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