OD KONUSNIH DO CILINDRIČNIH KARTOGRAFSKIH PROJEKCIJ

FROM CONIC TO CYLINDRICAL MAP PROJECTIONS

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IZVLEČEK

V knjigah in učbenikih o kartografskih projekcijah se cilindrične, konusne in azimutne projekcije običajno obravnavajo ločeno. Včasih se omenja, da je mogoče cilindrične in azimutne projekcije interpretirati kot robne primere konusnih, vendar je to redko dokazano. Cilj tega članka je natančno in sistematično pokazati, kako na splošno pristopiti k reševanju problema prehoda iz konusne v ustrezno cilindrično projekcijo. Članek opozarja, kako je J. H. Lambert že leta 1772 dokazal, da je konformna cilindrična projekcija ustvarjena iz konformne konusne projekcije. Na podlagi te zamisli je prikazano, kako je mogoče iz pokončnih konusnih izpeljati ne le konformne, temveč tudi ekvivalentne (enakoploščinske) in ekvidistantne (enakodolžinske) cilindrične kartografske projekcije. Čeprav se zdi, da prispevek obravnava precej dobro znano in intuitivno lastnost konusnih projekcij, bo tudi pokazal, da prehod iz konusne v ustrezno cilindrično projekcijo ni vedno mogoč.

ABSTRACT

In books and textbooks on map projections, cylindrical, conic and azimuthal projections are usually considered separately. It is sometimes mentioned that cylindrical and azimuthal projections can be interpreted as limiting cases of conic, but this is rarely proven. The goal of this article is to show in a rigorous and systematic way how to generally approach solving the problem of transition from a conic to a corresponding cylindrical projection. This article points to the fact that J. H. Lambert showed as early as 1772 that a conformal cylindrical projection is created from a conformal conic projection. Following his idea, this paper shows that not only conformal, but also equal-area and equidistant cylindrical projections can be derived from corresponding conic map projections. Although it seems that the paper deals with quite well known and intuitive property of conic projections, it will also show that the transition from the conic to the corresponding cylindrical projection is not always possible.

KLJUČNE BESEDE

kartografska projekcija, konusna projekcija, cilindrična projekcija

KEY WORDS

map projection, conic projection, cylindrical projection

1 MOTIVATION AND INTRODUCTION

In books and textbooks on map projections, cylindrical, conic and azimuthal projections are usually considered separately, and sometimes it is mentioned that cylindrical and azimuthal projections can be interpreted as limiting cases of conic projections (Lee, 1944; Kavrayskiy, 1958, 1959; Jovanović, 1983; Vakhrameyeva et al., 1986; Snyder, 1987; Kuntz, 1990; Canters, 2002; Monmonier, 2004; Serapinas, 2005), but it is rarely attempted to prove (Lambert, 1772; Hinks, 1912; Hoschek, 1969; Daners, 2012). This fact was the motivation for the research that preceded the writing of this article.

Lambert (1772) finished his derivation of the transformation from the conformal conic projection to the Mercator projection without any explanation of the connecton of his final expression and the Mercator projection.

Hinks (1912) uses Lambert's approach for the construction of cylindrical projections as the limiting cases of a conic projections. For a simple equal-area projection, with one standard parallel he missed to give a derivation.

Hoschek (1969) derives the equations of cylindrical equidistant, equal-area and conformal projections as limiting cases of corresponding conic projections, whereby he interprets conic projections as projections of a sphere onto a cone, not into a plane. In doing so, he uses expansions in the Taylor series, but when neglecting terms of a higher order, he makes the mistake of assuming that the indefinite expression $0 \cdot \infty$ is equal to 0, which in general does not have to be the case.

Daners (2012) derives the Mercator projection equations as the limiting case of a conformal conic projection of a sphere onto a cone that touches the sphere along a parallel. Daners does not deal with equidistant, equal-area or other projections.

The goal of this article is to show in a rigorous and systematic way how to generally approach solving the problem of transition from a conic to a corresponding cylindrical projection. We will also show that the transition from the conic to the corresponding cylindrical projection is not always possible. The transition from the conical to the corresponding azimuth projection is direct, there are no problems or open questions with it, so we will not deal with it.

The following section demonstrates Lambert's derivation of the Mercator projection as a special case of the conformal conic projection which is further explained in the section 3. Section 4 presents generic derivations of cylindrical as special case of conic projections having conformal, equal-area and equidistant properties. Final section presents concluding remarks.

2 LAMBERT'S DERIVATION OF THE MERCATOR PROJECTION AS A SPECIAL CASE OF THE CONFORMAL CONIC PROJECTION

The beginnings of the theory of mapping one surface to another belong to Johann Heinrich Lambert (1728–1777), who dealt with the general problem of mapping a sphere and an ellipsoid into a plane in the chapter *Anmerkungen und Zusätze zur Entwerfung der Land und Himmelscharten* (Notes and additions to the establishment of maps of the Earth and the Sky) printed in to the third part of his *Beyträge zum Gebrauche der Mathematik und deren Anwendung* (Contributions to the use of mathematics and its application) (Lambert, 1772).

Lambert was the first mathematician who dealt with the general properties of map projections. He first considered the properties of conformality and equivalence and pointed to the fact that these two properties are mutually exclusive. In the aforementioned book, he published seven new map projections that he did not name, but today they are known as:

1. The Lambert conformal conic
2. The transverse Mercator
3. The Lambert azimuthal equal-area
4. The Lagrange projection
5. The Lambert cylindrical equal-area
6. The transverse cylindrical equal-area
7. The Lambert conic equal-area

The Lambert conformal conic projection is one of the most famous map projections. It is still used today in many countries. E.g. in Croatia, it is the official projection for topographic maps of smaller scales.

This projection was proposed by Lambert in his *Notes and Supplements to the Establishment of Earth and Sky Maps* published 250 years ago. The fourth subsection is entitled *A more general method of representing a spherical surface so that all angles preserve their sizes*. That subsection is further divided into paragraphs and begins with §47 in which Lambert writes that the stereographic representation of the spherical surface, as well as Mercator nautical charts, have the property that all angles retain the size they had on the surface of the globe. This gives the greatest resemblance that an arbitrary small figure can have in the plane to that drawn on the surface of the sphere.

Until then, the question of whether this property appears only in the two mentioned projections or whether it can be reached from one to the other, whose representations are so different, can be reached with the help of intermediate steps. In §48 Lambert derives the formula for the conformal projection of the unit sphere and arrives at the equation $x = (\tan\frac{1}{2}\varepsilon)^m$, i.e. with today's usual notation

$$\rho = \tan^m(45° - \frac{\varphi}{2}), \tag{1}$$

where φ is latitude, m parameter, $0 < m < 1$, and ρ polar radius in the polar coordinate system in the plane of projection. In addition to (1), the equation (2) should also be written

$$\theta = m\lambda, \tag{2}$$

where λ is longitude, and θ polar angle in the polar coordinate system in the plane of projection. Equations (1) and (2) define the Lambert conformal conical projection of the unit sphere for $0 < m < 1$. For $m = 1$ it is a polar aspect stereographic projection (conformal azimuthal projection) (Snyder, 1987; Peterca, 2001; Frančula, 2004). However, for $m = 0$ it is not clear at first glance that it will be the Mercator projection (normal aspect conformal cylindrical projection). In fact, for $m = 0$, (1) and (2) give $\rho = 1$ and $\theta = 0$, and these are obviously not the equations of the Mercator projection. Here's how Lambert approaches it. In §50 it is written (with today's notation):

For the Mercator projection, $m = 0$. This only gives $\rho = 1$. We can write

$$\rho = \tan^m\left(45° - \frac{\varphi}{2}\right) = \left(\frac{1 - \tan\frac{\varphi}{2}}{1 + \tan\frac{\varphi}{2}}\right)^m$$

which gives

$$\rho = \tan^m\left(1 - m\tan\frac{\varphi}{2} + m\frac{m-1}{2}\tan^2\frac{\varphi}{2} - etc.\right)\left(1 - m\tan\frac{\varphi}{2} + m\frac{m+1}{2}\tan^2\frac{\varphi}{2} - etc.\right)$$

so that for $m = 0$ it is obtained

$$\frac{1-\rho}{2m} = \tan\frac{\varphi}{2} + \frac{1}{3}\tan^3\frac{\varphi}{2} + \frac{1}{5}\tan^5\frac{\varphi}{2} + \frac{1}{7}\tan^7\frac{\varphi}{2} + etc. = \frac{1}{2}\ln\cot\left(45° - \frac{\varphi}{2}\right).$$

Here $\frac{1-\rho}{m}$ shows the degrees calculated from the equator and they increase proportionally with $\ln\cot\left(45° - \frac{\varphi}{2}\right) = \ln\tan\left(45° + \frac{\varphi}{2}\right)$. This concludes Lambert's derivation for the Mercator projection.

3 THE MERCATOR PROJECTION AS A SPECIAL CASE OF THE LAMBERT CONFORMAL CONIC PROJECTION

The first thing we can notice is that every map projection is determined by two equations, usually in the form

$$x = x(\varphi, \lambda),\ y = y(\varphi, \lambda), \tag{3}$$

where φ, λ are latitude and longitude, respectively. For normal aspect cylindrical projections, such as the Mercator, the equations are simpler

$$x = x(\lambda),\ y = y(\varphi), \tag{4}$$

if we have chosen a mathematical coordinate system. However, in deriving the formula for the Mercator projection, Lambert does not mention the expression for the function $x = x(\lambda)$. He probably thought that $x = \lambda$ was too simple or obvious to not mention it. This equation can be derived from the Cauchy -Riemann conditions for conformal mappings, which he probably knew about because they were first used by d'Alembert (1717–1783). Regardless of the aforementioned, in §71 from the same Lambert book, we can read that the Mercator projection is described by the equations

$$x = \lambda,\ y = \ln\tan\left(45° + \frac{\varphi}{2}\right). \tag{5}$$

Lambert finished his derivation of the transformation from the conformal conic projection to the Mercator projection without comment or explanation about the connection between the limiting value of the fraction $\frac{1-\rho}{m}$ and the Mercator projection. So, we will do that in this section.

Let us first notice that when Lambert says that he will calculate $\frac{1-\rho}{m}$ for $m = 0$, he actually means the limiting value of that fraction when $m \to 0$. In doing so, he uses series development, which was common for him and at that time. Also, he does not use limes label for calculation the limit value, as is common nowadays. Instead of using series development, it would be simpler to see that when $m \to 0$ then $\rho \to 1$. This means that $\lim\limits_{m\to 0} = \frac{1-\rho}{m}$ is of the form $\frac{0}{0}$, so it can be solved by applying the l'Hôpital rule:

$$\lim_{m\to 0}\frac{1-\rho}{m} = \lim_{m\to 0}\frac{-\frac{d\rho}{dm}}{1} = -\lim_{m\to 0}\tan^m\left(45°-\frac{\varphi}{2}\right)\ln\tan\left(45°-\frac{\varphi}{2}\right) = \ln\cot\left(45°-\frac{\varphi}{2}\right) = \tag{6}$$
$$= \ln\tan\left(45°+\frac{\varphi}{2}\right).$$

We can reach the same result, but without applying l'Hôpital's rule, if we observe that $\ln\rho$ can be developed in the series

$$\ln\rho = \rho - 1 - \frac{(\rho-1)^2}{2} + \frac{(\rho-1)^3}{2} - \cdots \tag{7}$$

and that $\ln\rho$ is closer to $\rho - 1$ the closer ρ is to 1. So,

$$\lim_{m\to 0}\frac{1-\rho}{m} = -\lim_{m\to 0}\frac{\ln\rho}{m} = -\lim_{m\to 0}\frac{\ln\tan^m\left(45°-\frac{\varphi}{2}\right)}{m} = -\ln\tan\left(45°-\frac{\varphi}{2}\right) = \ln\tan\left(45°+\frac{\varphi}{2}\right). \tag{8}$$

In order to better understand why Lambert considered the fraction $\frac{1-\rho}{m}$, we will use the explanation of Hinks (1912). On page 70 of his book, Hinks (1912) states that a normal aspect cylindrical projection is a special case of a normal aspect conic projection when the pole of the projection goes to infinity. Parallels then become straight lines, ρ is infinite for all parallels, m is zero. Formulas for conic projections should be adapted to give not ρ_0 and ρ, but $\rho_0 - \rho$, the distance between any chosen parallel and the standard parallel. On page 103 of the same book, he examines the Mercator projection in more detail. It begins with the general case of a conformal conic projection with one standard parallel (in our notation and with a sphere radius equal to 1):

$$\rho = K\tan^m\left(45°-\frac{\varphi}{2}\right) \tag{9}$$

with

$$K = \frac{\cos\varphi_0}{m\tan^m\left(45°-\frac{\varphi}{2}\right)}. \tag{10}$$

If our cone becomes a cylinder tangent to the sphere along the equator, it is clear that K and ρ become infinitely large, and that the cone constant m is equal to zero. But in that case, it is

$$Km = \frac{\cos 0}{\tan^0(45°)} = 1. \tag{11}$$

Furthermore,

$$\rho_0 = \rho(\varphi_0) = K. \tag{12}$$

Now we can write

$$\rho_0 - \rho = K - K\tan^m\left(45° - \frac{\varphi}{2}\right) = K\left[1 - \tan^m\left(45° - \frac{\varphi}{2}\right)\right] = Km\frac{1 - \tan^m\left(45° - \frac{\varphi}{2}\right)}{m} = \tag{13}$$
$$= \frac{1 - \tan^m\left(45° - \frac{\varphi}{2}\right)}{m}$$

and due to (8) and (5) we have

$$\lim_{m\to 0}(\rho_0 - \rho) = \lim_{m\to 0}\frac{1 - \tan^m\left(45° - \frac{\varphi}{2}\right)}{m} = \ln\tan\left(45° + \frac{\varphi}{2}\right). \tag{14}$$

Since the equations of the normal aspect Mercator projection are (5) then we clarified the result of Lambert's approach to the Mercator projection (conformal cylindrical) as a special case of the conformal conic.

4 CYLINDRICAL PROJECTION AS A SPECIAL CASE OF CONIC PROJECTION – GENERAL CASE

A more general question now arises: how we from some normal aspect conic projection given by the equations in the polar coordinate system in the plane of the projection

$$\theta = m\lambda,\ \rho = \rho(\varphi) \tag{15}$$

could derive the equations of a normal aspect cylindrical projection in a rectangular system in the projection plane

$$x = n\lambda,\ y = y(\varphi) \tag{16}$$

which will have the same properties as the conic given by equations (15)? For example, if the equal-area conic projection is given by equations (15), what are the equations (16) of the corresponding equal-area cylindrical projection. Or vice versa, if the equal-area cylindrical projection is given by equations (16), what are the equations of corresponding equal-area conic projections? We will give the answer in this section.

Let the normal aspect conic projection be given by the equations in the polar coordinate system in the plane of projection (15) where φ and λ are latitude and longitude, respectively, on the unit sphere, m is a parameter, $0 < m < 1$. It is known from the literature that the local linear scale factors along the meridian and parallel are (Peterca, 2001; Frančula, 2004)

$$h = -\frac{d\rho}{d\varphi},\ k = \frac{m\rho}{\cos\varphi}. \tag{17}$$

Furthermore, the equations of the normal aspect cylindrical projection in the rectangular coordinate system in the projection plane are (16) where φ and λ are latitude and longitude, respectively, on the unit sphere, n is a parameter, usually $n > 0$. It is known from the literature that the local linear scale factors along the meridian and parallel are (Peterca, 2001; Frančula, 2004)

$$h = \frac{dy}{d\varphi}, k = \frac{dx}{\cos\varphi d\lambda} = \frac{n}{\cos\varphi}. \tag{18}$$

4.1 Conformal projections

Let $h = k$, i.e. we are working with conformal mappings. From (17) it follows

$$-\frac{d\rho}{d\varphi} = \frac{m\rho}{\cos\varphi}, \tag{19}$$

or

$$-\frac{d\rho}{\rho} = \frac{md\varphi}{\cos\varphi}. \tag{20}$$

Similarly, from (18) it follows

$$\frac{dy}{d\varphi} = \frac{n}{\cos\varphi}, \tag{21}$$

or

$$dy = \frac{nd\varphi}{\cos\varphi}. \tag{22}$$

By comparing relations (20) and (22) we have

$$dy = -\frac{n}{m}\frac{d\rho}{\rho} \tag{23}$$

and from there after integration

$$y = -\frac{n}{m}\ln\rho + const. \tag{24}$$

Vice versa,

$$\frac{d\rho}{\rho} = -\frac{m}{n}dy \tag{25}$$

so it is after integration

$$\ln\rho = -\frac{m}{n}y + const. \tag{26}$$

which is equivalent to (24).

If we substitute Lambert's ρ from (1) in (24), we get

$$y = -\frac{n}{m}\ln\rho = -n\ln\tan\left(45° - \frac{\varphi}{2}\right) = n\ln\tan\left(45° + \frac{\varphi}{2}\right), \tag{27}$$

where we chose $y = 0$ for $\varphi = 0$, i.e. *const.* = 0. For all normal aspect cylindrical projections, it is (Peterca, 2001; Frančula, 2004)

$$x = n(\lambda - \lambda_0). \tag{28}$$

Thus, by applying formula (24), we obtained the equation for $y = y(\varphi)$ of the normal aspect Mercator projection directly from the equation for $\rho = \rho(\varphi)$ of the normal aspect Lambert conformal projection. Equations (27) and (28) are the equations of the normal aspect Mercator projection of the sphere (Snyder, 1987; Peterca, 2001; Frančula, 2004).

4.2 Equal-area projections

Let $hk = 1$, i.e. we are working with equal-area mappings. From (17) it follows

$$-\frac{d\rho}{d\varphi}\frac{m\rho}{\cos\varphi} = 1, \tag{29}$$

or

$$-\rho d\rho = \frac{\cos\varphi d\varphi}{m}. \tag{30}$$

Similarly, from (18) follows

$$\frac{dy}{d\varphi}\frac{n}{\cos\varphi} = 1, \tag{31}$$

or

$$dy = \frac{\cos\varphi d\varphi}{n}. \tag{32}$$

By comparing relations (30) and (32) we have

$$dy = -\frac{m}{n}\rho d\rho, \tag{33}$$

and from there after integration

$$y = -\frac{m}{n}\frac{\rho^2}{2} + const. \tag{34}$$

Vice versa,

$$\rho d\rho = -\frac{n}{m}dy \tag{35}$$

so it is after integration

$$\frac{\rho^2}{2} = -\frac{n}{m}y + const. \tag{36}$$

which is equivalent to (34).

By solving the differential equation (30), we can obtain the equation of the normal aspect conic equal-area projection (Lapaine and Triplat Horvat, 2014)

$$\rho = \sqrt{C - \frac{2}{m}\sin\varphi}, \tag{37}$$

where C is a constant. If we substitute that ρ in (26), we get

$$y = -\frac{m}{n}\frac{\rho^2}{2} + const. = \frac{\sin\varphi}{n}, \tag{38}$$

where we chose $y = 0$ for $\varphi = 0$, i.e. $const. = \frac{Cm}{2n}$. The equation (28) remains to be valid also for the normal aspect equal-area cylindrical projection.

Thus, by applying formula (34), we obtained the equation $y = y(\varphi)$ of the normal aspect equal-area cylindrical projection directly from the equation $\rho = \rho(\varphi)$ of the normal aspect equal-area conic projection. Equations (38) and (28) are the equations of the normal aspect equal-area cylindrical projection of the sphere (Snyder, 1987; Frančula, 2004).

4.3 Equidistant projections along meridians

Let $h = 1$, i.e. we are working with equidistant mappings along meridians. It follows from (17)

$$-\frac{d\rho}{d\varphi} = 1, \tag{39}$$

or

$$-d\rho = d\varphi. \tag{40}$$

Similarly, from (18) it follows

$$\frac{dy}{d\varphi} = 1, \tag{41}$$

or

$$dy = d\varphi. \tag{42}$$

By comparing relations (40) and (42) it is not difficult to find a connection

$$dy = -d\rho \tag{43}$$

and from there after integration

$$y = -\rho + const. \tag{44}$$

Vice versa,

$$d\rho = -dy \tag{45}$$

so it is after integration

$$\rho = -y + const. \tag{46}$$

which is equivalent to (44).

By solving the differential equation (40), we can obtain the equation of the normal aspect conic projection equidistant along meridians

$$\rho = C - \varphi, \tag{47}$$

where C is a constant. If we substitute that ρ in (44) we can get

$$y = -\rho + const. = \varphi \tag{48}$$

where we chose $y = 0$ for $\varphi = 0$, i.e. $const. = C$. Thus, by applying formula (44), we obtained the equation of the normal aspect cylindrical projection equidistant along meridians directly from the equation of the normal aspect conic projection equidistant along meridians. Equations (48) and (28) are the equations of a normal aspect cylindrical projection equidistant along meridians (Snyder, 1987; Frančula, 2004).

4.4 Equidistant projections along parallels

Let k=1, i.e. we are working with equidistant mappings along parallels. It follows from (17)

$$\frac{m\rho}{\cos\varphi} = 1, \tag{49}$$

or

$$\rho = \frac{\cos\varphi}{m}, \tag{50}$$

Such a projection is not found in cartographic literature, except for m=1 when it comes as the orthographic polar projection. Thus, we can consider it a generalization of the orthographic projection. From (18) it follows

$$\frac{n}{\cos\varphi} = 1, \tag{51}$$

or

$$n = \cos\varphi. \tag{52}$$

Since n is a constant quantity, and (52) should be valid for all values of φ, it is immediately clear that from a normal aspect conic projection equidistant along parallels it is not possible to obtain a normal aspect cylindrical projection equidistant along parallels, because such does not exist.

5 CONCLUSION

Johann Heinrich Lambert (1772) derived the equations of a normal aspect conformal conic projection and showed how these equations can be used to get the equations of a normal aspect conformal cylindrical (Mercator) projection. In this paper Lambert's derivation is discussed and further explained. This gave the motivation to investigate the possibility of transition from some conic to corresponding cylindrical projection with the same properties (conformality, equal-area, equidistance). In this sense, the appropriate formulas for conformal, equal-area and equidistant projections along meridians are derived. In addition, it was shown that an analogous procedure is not possible for projections equidistant along parallels.

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