RECENZIRANI ČLANKI | PEER-REVIEWED ARTICLES

ZNAČILNOSTI VREDNOTENJA NEPREMIČNIN Z UPORABO ALTERNATIVNEGA PRISTOPA S PSEVDOINVERZIJO MATRIKE

FEATURES OF DETERMINING THE REAL PROPERTY VALUATION USING THE ALTERNATIVE APPROACH OF MATRIX PSEUDO-INVERSION

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IZVLEČEK

V delu je predstavljena uporaba metode psevdoinverzije matrike za določitev tržne vrednosti nepremičnin z uporabo matematičnega pristopa metode primerljivih prodaj podobnih nepremičnin. Uporaba sodobnih matematičnih metod za izračun vrednosti nepremičnin nam omogoča, da odpravimo vpliv subjektivnosti cenilca. Eden od glavnih ciljev raziskovanja in analize dejavnikov prilagajanja je analiza trga nepremičnin, strukture in porazdelitve stroškov za oblikovanje sistema izboljšav nepremičnin in elementov infrastrukturne opreme naselja. Metoda psevdoinverzije matrike za določitev tržne vrednosti nepremičninskih objektov je bila prvič uporabljena za primer, ko ni bilo mogoče dobiti inverzne matrike na klasičen način. Izvedene študije z uporabo teorije psevdoinverzne matrike omogočajo tudi določitev koeficientov za vrednost predmeta vrednotenja glede na dejavnike, ki najmočneje vplivajo na ceno. Rezultat, pridobljen s to metodo, lahko uporabimo kot podlago za primerjavo z vrednostmi, ki so določene s klasičnimi metodami vrednotenja.

ABSTRACT

The purpose of the work is the application of using the method of matrix pseudo-inversion to determine the market value of real estate objects using the mathematical apparatus of the method of comparing sales of similar real estate objects. Using modern mathematical methods to calculate the value allows us to overcome the influence of the appraiser's subjectivity. One of the main goals of research and analysis of adjustment factors is the analysis of the real estate market, the structure and distribution of costs for creating a system of real estate improvements and components of the settlement's infrastructure. For the first time, the method of matrix pseudo-inversion was applied to determine the market value of real estate objects for the case when it is impossible to obtain the inverse matrix in the classical way. Conducted studies using the pseudo-inverse matrix theory also make it possible to determine the coefficients to the value of the object of evaluation according to the most influential price-forming factors. The result obtained by this method can serve as a benchmark when the value is determined by the classical valuation methods.

KLJUČNE BESEDE

tržna vrednost, dejavniki vpliva na ceno, psevdoinverzija matrike, analogni objekt, trg nepremičnin, digitalizacija, množično vrednotenje

KEY WORDS

market value, price-forming factors, pseudo inverse matrix, analogue object, real estate market, digitisation, mass land valuation.

1 METHODS

Sales matching is a method of determining market value by analysing the prices of sales agreements or offers for the sale of objects similar to the appraised object that took place on the market shortly before the appraisal date. The sales matching method models the real estate market based on transaction prices by comparing appraised objects to analogue objects that have recently sold. It is evident that under the typical conditions of the agreements, the sale prices (offers) of objects as identical as possible in terms of characteristics to the evaluated object will serve as an indicator of its market value (Hubar et al.,2020; Drapikovsky et al., 2016).

The key point in the method is the delineation of elements of comparison and the justification of corrections for differences between objects, as it allows to determine of the most significant features from the point of view of the market, which affect the value of the object of assessment. At the same time, the appraiser must answer the question: “How much would an analogous land plot be sold for if it had the characteristics of the land plot being evaluated?”. Suppose the analogue land plot’s beneficial properties exceed the land plot’s corresponding parameters being evaluated. In that case, its price must be reduced, and if they are inferior, its price must increase. That is, there is a process of replacing the characteristics of similar land plots with the features of the land plot being evaluated, and the sale price of the compared land plot is adjusted accordingly. The result of applying this methodological approach is the market value of the land plot, which is determined by transforming the sale prices of similar land plots into the market value of the assessed land plot. The conclusion about the final value of the object of assessment is not just the result of a calculation but the result of a particular cause-and-effect analysis. The total value of the land plot is both the minimum possible price for which the buyer can purchase it and the maximum possible price for which the seller is willing to sell it on an open competitive market, taking into account the economic situation at the time of sale (Hubar, 2020; Drapikovsky et al., 2016).

In the articles (Fys et al., 2021; Hubar. et al., 2020), the authors consider the influence of adjustment factors on determining the value of real estate. The vast majority of existing real estate valuation methods, based on defining and comparing the market value of real estate objects, are oriented to consider these factors. In the article (Hubar et al., 2020), research established the most important factors affecting the appraised value of the real estate; several main groups of factors that determine the greatest impact on the value of a real estate object are highlighted.

The application of the method of comparison of sales involves:

1. Identification of the object of evaluation and related rights.
2. Analysis of market information to identify transactions regarding real estate objects that are most similar to the evaluated object.
3. Delineation of elements of comparison that affect the value of value and reflect differences like transactions and characteristics of the object of evaluation and the objects selected for comparison.
4. Selection of relevant units of comparison.
5. Adjust sales prices for each object per existing differences between it and the object of evaluation.
6. Analysis of adjusted prices and similar objects to determine the value or range of values of the value of the evaluated object.

Using modern mathematical methods to calculate the value in the sales comparison method allows for overcoming the influence of the appraiser's subjectivity. The choice of one method or another depends on the amount and quality of the initial information about analogue objects. In addition, there is a direct relationship between the number of factors on which the appraiser compares and the number of similar objects. One of the main goals of research and analysis of adjustment factors is analysing the structure and distribution of costs for creating a system of real estate improvements and infrastructure components of the settlement. Only a detailed analysis of the impact of these factors will make it possible to assess the results' reliability (Drapikovsky et al., 2016).

The below formula determines the value of the real estate object according to this method:

$$C_L = C_a + \sum_{j=1}^{k} \Delta C_{aj}, \tag{1}$$

where C_L - the real estate value, determined by comparing the sale price of similar objects; C_a - the sale price of a similar real estate object; k - the number of comparison factors; ΔC_{aj} - the difference (correction) in the price (±) of the sale of similar real estate objects in comparison with the object of assessment by the j-th comparison factor.

To carry out the research, an essential condition must be met (Hubar Yu., 2020): the number of selected analogue objects must exceed the number of adjustment factors per unit, i.e.:

$$n = k + l, \tag{2}$$

where n - the number of analogy objects.

To perform calculations by formula (1), it is necessary to successively compare the object of assessment with each of the selected objects of analogues, resulting in a system n of linear equations:

$$\begin{cases} C_L = C_1 + \Delta C_{11} + \Delta C_{12} + \ldots + \Delta C_{1k} \\ C_L = C_2 + \Delta C_{21} + \Delta C_{22} + \ldots + \Delta C_{2k} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_L = C_n + \Delta C_{n1} + \Delta C_{n2} + \ldots + \Delta C_{nk} \end{cases}. \tag{3}$$

Since the adjustment coefficient depends on the differences (ΔC_{aj}) in the j-th price-forming factor between the object of evaluation and the a-th analogue, it is advisable to determine these coefficients as follows:

$$\Delta C_{aj} = (x_{0j} - x_{aj}) \cdot \Delta c_j = \Delta x_{aj} \cdot \Delta c_j, \tag{4}$$

where x_{0j} is the value of the j-th adjustment factor for the evaluation object; x_{aj} - the value of the j-th correction factor for the a-th analogy; Δc_j - contribution to the unit cost of the j-th adjustment factor.

Taking (4) into account, system (3) can be written as follows:

$$\begin{cases} C_L = C_1 + \Delta x_{11} \cdot \Delta c_1 + \Delta x_{12} \cdot \Delta c_2 + \ldots + \Delta x_{1k} \cdot \Delta c_k \\ C_L = C_2 + \Delta x_{21} \cdot \Delta c_1 + \Delta x_{22} \cdot \Delta c_2 + \ldots + \Delta x_{2k} \cdot \Delta c_k \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_L = C_n + \Delta x_{n1} \cdot \Delta c_1 + \Delta x_{n2} \cdot \Delta c_2 + \ldots + \Delta x_{nk} \cdot \Delta c_k \end{cases}. \tag{5}$$

In the system of linear equations (5), the values and (j = 1,..., k) are unknown.

For ease of solution, it is advisable to write the system of linear equations (5) in the following form:

$$\begin{cases} C_L - \Delta x_{11} \cdot \Delta c_1 - \Delta x_{12} \cdot \Delta c_2 - \ldots - \Delta x_{1k} \cdot \Delta c_k = C_1 \\ C_L - \Delta x_{21} \cdot \Delta c_1 - \Delta x_{22} \cdot \Delta c_2 - \ldots - \Delta x_{2k} \cdot \Delta c_k = C_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_L - \Delta x_{n1} \cdot \Delta c_1 - \Delta x_{n2} \cdot \Delta c_2 - \ldots - \Delta x_{nk} \cdot \Delta c_k = C_n \end{cases} . \quad (6)$$

The set of equations (6) is a system of n linear equations with $n = k + l$ variables since we initially assumed, by condition (2), that the number of analogue objects should exceed the number of analysed price-forming factors by one unit.

The solution of system (6) can be found using any of the generally available methods of linear algebra; however, in our opinion, it is more convenient to find the solution of this system using the matrix method:

$$\Delta X C_L = C_a, \quad (7)$$

where C_L - the resulting matrix presents the specific market value of the object of assessment and corrections to its value based on the previously established most influential factors; ΔX- matrix of coefficients for variables; C_a - a column vector of free terms.

$$\Delta X = \begin{bmatrix} 1 & -\Delta x_{11} & \ldots & -\Delta x_{1k} \\ 1 & -\Delta x_{21} & \ldots & -\Delta x_{2k} \\ \ldots & \ldots & \ldots & \ldots \\ 1 & -\Delta x_{n1} & \ldots & -\Delta x_{nk} \end{bmatrix}, C_L = \begin{bmatrix} C_L \\ \Delta c_1 \\ \ldots \\ \Delta c_k \end{bmatrix}, C_a = \begin{bmatrix} C_1 \\ C_2 \\ \ldots \\ C_n \end{bmatrix}. \quad (8)$$

The solution of system (7) exists in the case when the determinant of the inverse matrix is not equal to zero and, therefore, the system has one solution (Kyrchei I. ed. al., 2022):

$$C_L = \Delta X^{-1} C_a, \quad (9)$$

where ΔX^{-1} - the inverse of a matrix to a matrix ΔX.

This method allows the expert to relatively quickly obtain the market value of the real estate, which will serve as a reference when applying other valuation methods approved by international real estate valuation standards.

However, in practice, situations often arise when it is impossible to enter the inverse matrix or the initial matrix needs to be better determined [Gantmacher F., 2010]. Accordingly, the solution of equation (9) does not exist or needs to be corrected. Applying the pseudo matrix rotation is advisable [Adi Ben-Israel, 2003].

When solving a system of equations, a situation arises when the determinant of the system is equal to zero. The explanation of this situation is as follows: the studied objects are dependent on each other, and therefore the further course of obtaining a solution consists in finding this dependence and getting a system of equations already with a smaller number of research objects. But it can be challenging to establish and apply these relationships. Therefore, one of the alternative approaches can be the solution of such systems with the help of pseudo-rotation of matrices. The mathematical apparatus for obtaining the inverse matrix is developed in detail in mathematics and is widely used in the processing of geodesic networks. It should be noted that its implementation is burdened by somewhat cumbersome, though simple, calculations. They can be eliminated using the rotation algorithm proposed by the Ukrainian mathematician M. S. Syavavko. (Syavavko M., 2000).

Suppose the determinant of the system is equal to zero or the number of equations is less than the number of variables. In that case, the system is ambiguous, and the question arises of choosing such values for which the problem would have a logical meaning. This leads to the "pseudo-solution" and "pseudo-inverse matrix" concept. The content of the problem can be formulated as a solution to a system of linear equations (Malik, S. B., & Thome, N., 2014):

$$AX - b = 0, \tag{10}$$

where

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,1} & \dots & a_{2,n} \\ \dots & \dots & \dots & \dots \\ a_{m,1} & a_{m,2} & \dots & a_{m,n} \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_m \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}, \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \dots \\ \lambda_m \end{bmatrix}^T \tag{11}$$

provided $\sum_{i=1}^{n} x_i^2 \to \min$. This statement leads to the conditional extremum problem with the Lagrange function (Nashed, M. Z., 2014; Wang, G., ed. al., 2018).

$$F(X,\lambda) = \sum_{i=1}^{n} x_i^2 + \lambda A = \sum_{i=1}^{n} x_i^2 + \sum_{i=j}^{m} \lambda_j \left(\sum_{i=1}^{n} \left[a_{j,i} x_i - b_j \right] \right), \tag{12}$$

Then the solution of system (10) can be represented as

$$X = A^{+} b, \tag{13}$$

where A^{+} - pseudo-inverse matrix determined by the algorithm described in (Gantmacher, 2010).

However, the implementation of this algorithm is associated with specific difficulties in its performance (search for minimum values, selection of linearly independent equations), so we will apply the method of finding the "pseudo-inverse matrix" (Syavavko M., 1996; Syavavko M., 2000).

We present the main provisions of this algorithm for a special system of equations:

$$A'X - b = 0,\ A' = A^{T}A,\ b' = A^{T}b,\ m = n,\ \det(A') = 0 (\det(A') \approx 0),\ r = rang(A') < n \tag{14}$$

The solution is found by a sequence of such matrix operations:

$$s_1 = Sp(A') = \sum_{i=1}^{n} a_{i,i}, E = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & 1 & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}, \tag{15}$$

$$\begin{cases} C_1 = C_0 A' + s_1 E, \quad s_2 = \dfrac{1}{2} Sp(C_1 A') \\ \ldots \\ C_{r-1} = C_{r-2} A' + s_{r-1} E, \quad s_{r-1} = \dfrac{1}{r-1} Sp(C_{r-2} A') \end{cases}, \tag{16}$$

$$s_r = \frac{1}{r} Sp(C_{r-1} A'), \tag{17}$$

$$X = \frac{1}{s_r} C_{r-1} b'. \tag{18}$$

If the system is not singular, then its solution coincides with the given solution according to the formula (18). Also, for ill-conditioned linear systems, the values of the unknown variables obtained from expressions (14) - (18) are stable to the initial conditions, which gives a correct definition of the solution. Therefore, the presented technique can be fully applied to this problem.

2 RESULTS OF INVESTIGATIONS

Commercial real estate land was selected for the practical implementation of the research presented above. The evaluation object is located 20 km from the regional centre; at a distance of 600 m to the reservoir; at a distance of 500 m to the forest area; presence (absence) of a garage (0- absence, 1- presence); the object is gasified, electrified and severed; the attractiveness of the architectural environment of the location of the evaluation object is at an average level (4 points on a 10-point scale). After research and analysis of this real estate market segment, 11 similar objects were selected according to their characteristics (object area, date of sale, physical parameters, etc.). To carry out research according to formula (2), it is necessary to choose 10 price-forming factors.

We have established 10 price-forming factors that most significantly influence the value of the object of evaluation, namely:

- distance to the boundary of the regional centre, km – a quantitative factor;
- distance to the reservoir, km – a quantitative factor;
- distance to the forest, km – a quantitative factor;
- presence (absence) of a garage is a quality factor;
- availability (absence) of gasification, electrification and sewerage is a quality factor;
- the attractiveness of the architectural environment is a quality factor;
- the presence (absence) of noise and vibration is a quality factor;
- distance to the border of the district centre, km – a quantitative factor;
- distance to the railway station, km – a quantitative factor;
- the size of the plot of land, acre - a quantitative factor.

The factors by which an appraiser compares different properties when searching for similar properties can be classified as qualitative or quantitative. A qualitative factor is a feature that characterises some property or state, as well as the presence or absence of a given property of a given object. A quantitative factor is a feature whose individual value, obtained from measurement, observation or calculation, is expressed by a certain number. To conduct research, we will use a scale of quality ratings from 1 to 10, since it allows us to consider the degree of difference and has the smallest possible standard deviation.

The peculiarity of this example is that the object of evaluation is only gasified, not electrified and does not have sewerage. In contrast, similar objects do not have the characteristics of this price-forming factor.

The initial data for determining the value of the object of assessment and corrections to this value by price-forming factors are presented in Table 1.

Table 1: Digitised values of price-forming factors and the cost of similar objects in conventional units

Name of the object	distance to the boundary of the regional centre, km	distance to the reservoir, km	distance to the forest, km	presence (absence) of a garage	presence (absence) of gasification, electrification and	the attractiveness of the architectural environment	the presence (absence) of noise and vibration	distance to the border of the district centre, km	distance to the railway station, km	the size of the plot of land, acre	the cost of similar objects is conditional units
Object of evaluation	**20**	**0,6**	**0,5**	**1**	**1**	**4**	**6**	**3,2**	**1,5**	**15**	**-**
Analogue 1	15	0,2	0,2	0	0	1	2	2,0	0,3	14	30002
Analogue 2	12	0,9	2,0	0	0	5	5	3,0	3,0	17	32000
Analogue 3	12	0,8	1,2	1	0	9	8	1,0	3,0	18	34000
Analogue 4	14	0,2	0,7	1	0	7	7	3,0	2,0	12	36004
Analogue 5	15	0,2	1,8	0	0	6	3	4,0	2,0	15	31995
Analogue 6	20	1,0	1,0	1	0	1	1	2,5	1,0	17	35009
Analogue 7	22	0,5	1,0	0	0	5	4	4,5	0,7	10	24000
Analogue 8	25	0,3	0,5	1	0	6	4	6,0	0,2	15	32005
Analogue 9	35	0,4	1,4	0	0	4	1	10,0	1,5	11	30000
Analogue 10	33	0,1	1,5	0	0	2	6	7,0	2,0	12	31004
Analogue 11	25	0,3	0,2	1	0	3	2	6,0	0,2	15	34000

So, let's present the original matrix ΔX (8) obtained from Table 1 and the column vector of free terms C_a (the cost of 1 hectare of land):

$$\Delta X = \begin{bmatrix} 1 & -5 & -0,4 & -0,3 & -1 & -1 & -3 & -4 & -1,2 & -1,2 & -1 \\ 1 & -8 & 0,3 & 1,5 & -1 & -1 & 1 & -1 & -0,2 & 1,5 & 2 \\ 1 & -8 & 0,2 & 0,7 & 0 & -1 & 5 & 2 & -2,2 & 1,5 & 3 \\ 1 & -6 & -0,4 & 0,2 & 0 & -1 & 3 & 1 & -0,2 & 0,5 & -3 \\ 1 & -5 & -0,4 & 1,3 & -1 & -1 & 2 & -3 & 0,8 & 0,5 & 0 \\ 1 & 0 & 0,4 & 0,5 & 0 & -1 & -3 & -5 & -0,7 & -0,5 & 2 \\ 1 & 2 & -0,1 & 0,5 & -1 & -1 & 1 & -2 & 1,3 & -0,8 & -5 \\ 1 & 5 & -0,3 & 0 & 0 & -1 & 2 & -2 & 2,8 & -1,3 & 0 \\ 1 & 15 & -0,2 & 0,9 & -1 & -1 & 0 & -5 & 6,8 & 0 & -4 \\ 1 & 13 & -0,5 & 1 & -1 & -1 & -2 & 0 & 3,8 & 1,5 & -3 \\ 1 & 5 & -0,3 & -0,3 & 0 & -1 & -1 & -4 & 2,8 & -1,3 & 0 \end{bmatrix}$$

$$C_a = \begin{bmatrix} 2143,00 \\ 1882,35 \\ 1888,89 \\ 3000,33 \\ 2133,00 \\ 2059,35 \\ 2400,00 \\ 2133,67 \\ 2727,27 \\ 2583,67 \\ 2266,67 \end{bmatrix}$$

It is impossible to rotate the original symmetric matrix using the classical method, and therefore we will use the alternative method of pseudo rotation of the matrix using formulas (14-18). So, we obtained the inverse matrix, which will have the following form:

$$\Delta X^{-1} = \begin{bmatrix} -0,091 & -0,014 & 0,006 & 0,029 & 0,008 & 1,355 & 5,053 & -0,113 & 0,595 & 0,013 & 1,020 \\ -0,053 & 0,218 & -0,354 & -0,172 & 0,147 & 0,789 & 1,543 & -0,268 & -0,747 & 0,1123 & 0,502 \\ -0,089 & -0,353 & 0,359 & 0,168 & -0,119 & 1,146 & 3,518 & 0,190 & 0,329 & -0,102 & 0,550 \\ 0,176 & -0,190 & -0,311 & -0,152 & 0,173 & -3,024 & -5,928 & 0,639 & -0,386 & 0,083 & -2,733 \\ 0,002 & -0,006 & 0,213 & 0,166 & -0,076 & 0,375 & -1,595 & -0,397 & 1,026 & -0,035 & 0,881 \\ 0,091 & 0,014 & -0,006 & -0,029 & -0,008 & -2,625 & -5,053 & 0,113 & -0,595 & -0,013 & -2,290 \\ 0,081 & 0,256 & 0,288 & 0,076 & -0,009 & 1,847 & 1,978 & -0,352 & -0,398 & -0,045 & 1,473 \\ -0,087 & 0,766 & -0,404 & -0,301 & -0,076 & -1,029 & -0,523 & -0,577 & -0,227 & 0,072 & -0,732 \\ 0,007 & -0,333 & -0,109 & 0,105 & -0,087 & -0,566 & 1,731 & 0,230 & -0,108 & -0,004 & -0,477 \\ -0,020 & 0,015 & 0,093 & -0,093 & 0,085 & 0,746 & -0,184 & -0,234 & 0,430 & -0,043 & 0,509 \\ -0,016 & -0,372 & 0,2261 & 0,203 & 0,120 & 1,238 & -0,540 & 0,769 & 0,081 & -0,038 & 0,549 \end{bmatrix}$$

Multiplying the inverse matrix, obtained by pseudo-rotation of the original matrix, with the matrix of free terms (results of completed sales of similar objects), we get the resulting matrix:

$$C_L = \begin{bmatrix} 3635,09 \\ 31,00 \\ -144,18 \\ 605,36 \\ -45,63 \\ -16,97 \\ 7,75 \\ -58,74 \\ -184,18 \\ -307,08 \\ 194,57 \end{bmatrix}$$

Therefore, the first element of the resulting matrix of 3635.09 conditional units can be considered the market value of 1 hundredth of the real estate valuation object on the relevant valuation date. As the area of the research object is 15 acres (table 1), the final value of the evaluation object will be 3635 conditional units x 15 acres = 54525 conditional units. Other elements of the resulting matrix reveal the quantitative influence of the ten price-forming factors presented above on the assessment of the market value of other real estate objects that the appraiser will evaluate.

3 DISCUSSION AND CONCLUSIONS

Other elements of the resulting matrix show the influence of price-forming factors on the change in the value of the object of evaluation and are interpreted as follows:

1. The object's value increases by 31,00 conditional units per 1 acre when the object is closer to the regional centre by 1 km.
2. The object's value decreases by 144,18 conditional units per 1 acre when the object is closer to the reservoir by 1 km.
3. The object's value increases by 605.36 conditional units per 1 acre when the object is closer to the forest by 1 km.
4. The object's value decreases by 45.63 conditional units per 1 acre when the object has a garage.
5. The object's value decreases by 16.97 conditional units per 1 acre when the object is in the presence of gasification, electrification and sewerage.
6. The object's value increases by 7.75 conditional units per 1 acre when the quantitative scale increases by one unit for the attractiveness of the architectural environment.
7. The object's value decreases by 58.74 conditional units per 1 acre when the quantitative scale increases by one unit due to the increase in the level of noise and vibration.
8. The object's value decreases by 184.18 conditional units per 1 acre when the object is closer to the district centre by 1 km.
9. The object's value decreases by 307.08 conditional units per 1 acre when the object is closer to the railway station by 1 km.
10. The object's value increases by 194.57 conditional units per 1 acre when the object's area increases by 1 acre.

Conducted studies using a pseudo-rotation matrix make it possible to determine the market value and correction coefficients to the value of the object of assessment in the case when the original matrix cannot be inverted by the classical method (there is no solution).

The main problems that assessors have to face when performing the procedures of this method are:

– the impossibility of verifying the fulfilment of all the conditions and requirements necessary for the price of the transaction for the sale of the real estate object to be considered as a market value;
– insufficient completeness of data on the characteristics of analogue objects;
– lack of skills in the practical use of statistical methods of processing and analysis of sample data;
– lack of a criterion for selecting analogue objects from the database, as there are many factors to be compared;
– lack of a generally recognised methodology for calculating corrections to the value of analogue objects.

The first two problems are challenging to solve because such information belongs to the category of trade secrets, and therefore, access to it is minimal. To solve the issues related to the processing and analysis of statistical information (the following two problems), it is possible to apply the methods of mathematical statistics using the procedure of correlation and regression analysis.

If there is a sufficiently large amount of data on similar objects, preference should be given to statistical methods that will allow:

- identify regularities against the background of coincidences;
- build multi-factor assessment models and obtain reasonable conclusions and forecasts.

So, we have received a reasonable and calculated evidentiary result of the assessment, which must be checked for compliance with common sense, both from the point of view of a potential seller or buyer, and from the point of view of a potential opponent, namely regulatory bodies and courts.

Contribution of the article to the profession. When performing real estate appraisal work, most appraisers are limited to an expert method based on professional experience and intuition, which is a subjective approach and causes a corresponding level of distrust in the final result of the real estate market value. With limited data on similar objects, the correct use of statistical methods becomes problematic. Therefore, applying the proposed method allows us to determine the market value of the object of assessment reasonably quickly and with essential mathematical evidence. The result obtained by this method can serve as a reference point for the market value during its determination by classical valuation methods. The art of assessment consists of the correctness of the choice of methodology, the use of representative initial data, the performance of correct calculations and the ability to convince all interested parties of the correctness and correctness of its results.

References:

Adi Ben-Israel, Thomas N.E. Greville (2003). Generalised Inverses. Theory and Applications. Springer. – 436p.

Drapikovsky O., Ivanova I., Tereshchenko N. (2016) Methods and models of real estate evaluation – Kyiv: PJSC "Vipol", 2016. – 512p.

Fys M., Balian A., Brydun A., Yurkiv M., Lozynskyi V. (2021) One method for solving ill-conditioned and degenerate systems of linear equations and its application for equalisation a levelling network GeoTerrace-2021, October 4-6, 2021, Lviv, Ukraine. DOI: https://doi.org/10.3997/2214-4609.20215K3022.

Gantmacher F. Matrix theory. (2010) — 5th. — M: Physical and mathematical literature, — 559 p.— ISBN 5-9221-0524-8.

Hubar Yu., Khavar Y., Sai V., Vinarchyk L., Ivakhniuk K. (2020) Investigation of rented immovable real estate right // International Conference of Young Professionals «GeoTerrace-2020», Lviv, Ukraine. DOI: https://doi.org/10.3997/2214-4609.20205748.

Hubar Yu. (2020) Geodetic support and improvement of real estate valuation methods and models: monograph – Lviv: Center of Europe. – 326 p.

Kyrchei, Ivan, ed. (2022) Generalized inverses: Algorithms and applications. Nova Science Publishers. DOI: https://doi.org/10.52305/MJVE4994.

Malik, S. B., & Thome, N. (2014). On a new generalised inverse for matrices of an arbitrary index. Applied Mathematics and Computation, 226, 575-580.

Nashed, M. Z. (Ed.). (2014). Generalised Inverses and Applications: Proceedings of an Advanced Seminar Sponsored by the Mathematics Research Center, the University of Wisconsin-Madison, October 8-10, No. 32. Elsevier.

Syavavko M. (1996) J-fractional regularisation of incorrect linear equations. – Lviv. - Vol. 48. – P.1130-1143.

Syavavko M., Rybytska O. (2000) Mathematical modelling under uncertainty. – Lviv: "Ukrainian technologies" – 320 p.

Wang, G., Wei, Y., Qiao, S., Lin, P., & Chen, Y. (2018). Generalised inverses: theory and computations (Vol. 53). Singapore: Springer. ISBN: 978-981-13-0146-9.

Hubar Y., Fys M., Brydun A., Sohor A. (2023). Features of determining the real property valuation using the alternative approach of matrix pseudo-inversion. Geodestski vestnik, 67 (3), 374-384.
DOI: https://doi.org/10.15292/geodetski-vestnik.2023.03.374-384

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