Testiranje uporabe metode simulacije Monte Carlo za določitev števila prostostnih stopenj v deformacijski analizi postopka München

Testing the use of the Monte Carlo simulation method to determine the number of degrees of freedom in the deformation analysis of the Munich approach

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IZVLEČEK

Pri statističnem testiranju hipotez preverjamo, ali testna statistika leži v območju, kjer hipoteze ne moremo zavrniti, pri čemer lahko kritične vrednosti izračunamo glede na znano porazdelitev verjetnosti. Empirični pristop za določitev ustreznega števila prostostnih stopenj v postopku München uporabimo za potrditev teoretično določenih vrednosti. Rezultati kažejo, da se uporabljena števila prostostnih stopenj ujemajo z empirično izračunanim številom pri statističnem testiranju hipotez pri testiranju skladnosti, pri testiranju sprememb dolžin in testiranju sprememb koordinat oglišč trikotnikov v geodetski mreži. Vendar pa se pri testiranju sprememb kotov med točkami v geodetski mreži empirična vrednost števila prostostnih stopenj razlikuje od števila, ki se običajno uporablja in je po našem mnenju pravilna vrednost za ustrezni statistični test. To štejemo za glavni prispevek tega dela.

ABSTRACT

When statistically testing the hypotheses, it can be checked if the test statistic lies in the fail-to-reject region, whose critical values can be computed according to the known probability distribution. The empirical approach to determine the appropriate value of the degrees of freedom is considered to confirm the theoretically defined values from the Munich approach. The results show that the commonly used numbers of degrees of freedom agree with the empirically derived numbers when statistically testing hypotheses in the congruence test, when testing the changes in distances and coordinates of the triangles' vertices. However, when testing the changes of angles between points in the geodetic network, the empirical value for the number of degrees of freedom is different from the number which is commonly applied. It can be considered the correct value for the corresponding statistical test, which is the main contribution of this work.

KLJUČNE BESEDE

simulacije Monte Carlo, število prostostnih stopenj, postopek München, statistično testiranje hipotez, deformacijska analiza geodetskih mrež

KEY WORDS

Monte Carlo simulation, number of degrees of freedom, Munich approach, statistical testing of hypotheses, deformation analysis of geodetic networks

1 Introduction

The Munich approach in deformation analysis is a geodetic statistical method that involves (as general property of all geodetic deformations' methods) the monitoring of displacements of points by selecting reference points on the surrounding terrain and control points on the observed object. According to statistical testing in the different methods of deformation analysis: Hannover approach (Pelzer, 1971), Delft approach (Heck et al., 1982), Karlsruhe approach (Heck, 1983), Fredericton approach (Chrzanowski, 1981) and Munich approach (Welsch, 1982, 1983; Welsch and Zhang, 1983) the adopted procedure is to reject null hypothesis about the displacements of points in geodetic networks.

The Munich approach was developed in Munich, Germany, and has been discussed in various geodetic publications (Welsch, 1982, 1983; Welsch and Zhang, 1983). The approach is a part of the broader field of deformation analysis, which is used in geodesy and geodynamics to study deformations of natural and man-made objects, crustal deformations and gravity field deformations. The method is based on statistical and geometrical analysis of deformation measurements, which is an important tool in engineering surveying (Caspary et al., 1990; Dermanis and Livieratos, 1983; Nowel, 2015).

Hypothesis testing in statistics is crucial for making decisions based on sample data. It helps in checking and controlling errors, ensuring scientific validity and making inferences about populations. In this study, the analysis of deformation will be adopted to explore the behaviour or deformed state of natural or man-made objects. In general, hypothesis testing is performed by the following procedure (Lehmann and Romano, 2022):

- The null H_0 and the alternative H_1 hypothesis about specific population property is set,
- The test statistics, which corresponds to the null hypothesis and for which we know probability distribution, is selected,
- The significance level α is selected and the limits of the reject region on the basis of α and the probability distribution of test statistic is determined,
- The sample data is used to compute the value of the test statistic,
- A conclusions are drawn:
  - If the value of the statistic is in the reject region, the null hypothesis is rejected at significance level α or
  - If the value of the statistic lies in the fail to reject region, the null hypothesis cannot be rejected at a significance level α.

When determining the probability distribution of the selected statistic, the limits of the reject region from the inverse cumulative distribution function can be computed. In some cases, the distribution of the statistics is Fisher's probability distribution (F-distribution). To be able to compute the limits of the reject region unambiguously, the correct number of degrees of freedom need to be known.

Displacements of unstable points can be identified with significance level α only if the precision of displacement is adequate. And in that context, the limit value that represents the lower limit of the displacement of unstable points, i.e. the minimum detectable deformation, is important.

In Munich approach of deformation analysis (Savšek and Ambrožič, 2023; Welsch, 1983; Welsch and Zhang, 1983):

- The test statistic for coordinate differences of all points in a two-dimensional geodetic network in two epochs is distributed according to the F-distribution $F_{f_\mathbf{u},f}$.
- The test statistic for changes in distance between two points P_i and P_j in two epochs is distributed according to the F-distribution $F_{1,f}$.
- The test statistic for changes in angles between points P_i, P_j and P_k in two epochs is distributed according to the F-distribution $F_{1,f}$.
- The test statistic for changes in coordinates of triangle vertices P_i, P_j and P_k in two epochs is distributed according to the F-distribution $F_{3,f}$.

where $f_\mathbf{u} = u - d$ is the number of degrees of freedom, $u = 2m$ is the number of coordinate unknowns in 2D network (m is the number of all points in geodetic network), d is the datum defect of geodetic network, $f = f_1 + f_2$ is the number of redundant measurements in adjustment of previous (f_1) and current (f_2) epoch or measurements. To compute critical values $F_{f_\mathbf{u},f}$, $F_{1,f}$ and $F_{3,f}$ correctly the correct number of degrees of freedom is needed.

To confirm the correct number of degrees of freedom in the computation of the critical values, Monte Carlo simulations can be used to determine the limits of the reject region, which is the main aim of this study. The procedure is based on the study by Savšek and Ambrožič, 2023, and in the following sequel the study represents the logical evolution of previous research.

2 Simulation of measurements and adjustment

The work started with the simulation of measurements in the geodetic network (i.e. of horizontal directions and distances) with software written for this purpose. The measurements are free of any gross or systematic errors, which means observations must have random errors only. Therefore, it can be assumed that the measurements are normally distributed around the mean value $\overline{y}_{meas}$ with standard deviation σ. To simulate measurements, a normally distributed random errors $u_k\sigma$ must be added to the expected values of measurements:

$$y_k = \overline{y}_{meas} + u_k\sigma, \tag{1}$$

where u_k is the standardized normally distributed random variable.

The expected values of the measurements are computed from the coordinates of the points, which are known as they are basis for simulations (Kuang, 1996).

Simulated k^{th} horizontal direction s_k (or bearing angle because in simulations the orientation of the initial direction at station point is arbitrary) in the triangulation geodetic network, which is obtained by the following equation:

$$s_k = \overline{s}_{koo} + u_{1k}\sigma_{hz}, \tag{2}$$

where $\overline{s}_{koo} = \arctan\dfrac{y_j - y_i}{x_j - x_i}$ is the bearing angle between points P_i and P_j, σ_{hz} is the standard deviation of horizontal direction, which is selected, and u_{1k} is standardized normally distributed random variable.

The simulated k^{th} horizontal distance d_k in geodetic trilateration network is obtained by the following equation:

$$d_k = \overline{d}_{koo} + u_{2k}\sigma_d, \tag{3}$$

where $\overline{d}_{koo} = \sqrt{\left(y_j - y_i\right)^2 + \left(x_j - x_i\right)^2}$ is the horizontal distance between points P_i and P_j, σ_d is the standard deviation of horizontal distance, which is selected and u_{2k} is standardized normally distributed random variable.

In the next step, these simulated measurements are adjusted using the Least Squares method (as if the measurements had actually been made) as constrained network where $\mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v}$ = min. or as free geodetic network where $\mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v}$ = min. and $\hat{\mathbf{x}}^{\mathrm{T}}\hat{\mathbf{x}}$ = min. (Kuang, 1996). The adjusted coordinates of new points in the network and the reference variance a posteriori s^2 are obtained.

```
Start
  • Read input data:
      • approximate coordinates of points $\mathbf{x}$
      • standard deviation of directions $\sigma_{hz}$
      • standard deviation of distances $\sigma_d$
  • For all simulations $c = 1, \ldots, Sim$:
      • For all measurements in the geodetic network $k = 1, \ldots, n$:
          • Generate samples of standardized normally distributed random variables:
              • $u_{1k}$ = randn, $u_{2k}$ = randn
          • Calculate measurements from approximate coordinates of points $P_i$ and $P_j$:
              • $\overline{s}_{koo} = \arctan \frac{y_j - y_i}{x_j - x_i}$
              • $\overline{d}_{koo} = \sqrt{\left(y_j - y_i\right)^2 + \left(x_j - x_i\right)^2}$
          • Compute simulated measurements:
              • $s_k = \overline{s}_{koo} + u_{1k}\sigma_{hz}$ … (2)
              • $d_k = \overline{d}_{koo} + u_{2k}\sigma_d$ … (3)
      • End of measurements in the geodetic network
      • Adjustment of simulated measurements in the geodetic network:
          • constrained network:                    • free network:
            $\mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v}$ = min.                           $\mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v}$ = min. and $\hat{\mathbf{x}}^{\mathrm{T}}\hat{\mathbf{x}}$ = min.
      • Results of adjustment:
          • adjusted coordinates of points $\hat{\mathbf{x}}_c$
          • cofactor matrix of coordinate unknowns $\mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c}$
          • a posteriori reference variance $s_c^2$
          • number of redundant measurements $f_c$
  • End of simulations
End
```

Figure 1: Flow chart of the simulations of measurements and adjustment.

After the described process of simulating measurements and adjustments, the results of the first iteration/simulation are obtained. The process of simulation of measurements and adjustments must be repeated, exactly as it has just been described, namely *Sim*-times, where *Sim* is the number of simulations. The result of this work of research is therefore the adjusted coordinates of the points of the geodetic network $\hat{\mathbf{x}}_c$ and the reference variances a posteriori s_c^2, where $c = 1, \ldots, Sim$. Since the geometry and precision of the measurements σ_{hz} and σ_d do not change between simulations, the corresponding cofactor matrix of coordinate unknowns $\mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c}$ in all simulations stays the same. Thus, in all iterations, also the cofactor matrix of the coordinate difference $\mathbf{Q}_\mathbf{u} = \mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c} + \mathbf{Q}_{\hat{\mathbf{x}}_{c+1}\hat{\mathbf{x}}_{c+1}} = 2\mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c}$, $c = 1, \ldots, (Sim - 1)$ stays the same as in (Savšek and Ambrožič, 2023).

3 Testing the transformation of the geodetic network

After iterations, the homogeneity of measurement accuracy is fulfilled and there are no non-identical points, since the simulations on the same geodetic network are repeated. Thus, between two simulations representing two times measurements (epochs), the a posteriori reference variance can be computed s^2 according to the equation in (Savšek and Ambrožič, 2023), which is given below:

$$s^2 = \frac{f_c s_c^2 + f_{c+1} s_{c+1}^2}{f_c + f_{c+1}}, c = 1, \ldots, (Sim - 1), \tag{4}$$

where f_c and f_{c+1} are the number of redundant measurements in two consecutive simulations and s_c^2 and s_{c+1}^2 are the reference variances a posteriori in two consecutive simulations.

3.1 Testing the congruence of geodetic network

Testing the congruence of the geodetic network, which is the next phase of the Munich approach, is not problematic, because test statistics between two simulations is generated (Welsch, 1983; Welsch and Zhang, 1983; Kuang, 1996):

$$T^2_{1_{c,c+1}} = \frac{s_\mathbf{u}^2}{s^2} = \frac{\mathbf{u}^\mathbf{T}_{c,c+1} \mathbf{Q}_\mathbf{u}^{-1} \mathbf{u}_{c,c+1}}{f_\mathbf{u} \cdot s^2}, c = 1, \ldots, (Sim - 1), \tag{5}$$

where $\mathbf{u}_{c,c+1}$ is the displacement vector of identical points between two simulations and $\mathbf{Q}_\mathbf{u}$ is the cofactor matrix of coordinate differences. The computation of the critical value $F_{f_\mathbf{u}, f}$ is straight forward as the degrees of freedom $f_\mathbf{u}$ and f are known.

3.2 Testing the changes of distances in geodetic network

When testing distance differences in the geodetic network, which is one of the components of testing the transformation of the geodetic network, test statistics between two simulations $T^2_{2D_{c,c+1}}$ (Welsch, 1982; Savšek and Ambrožič, 2023) is created:

$$T^2_{2D_{c,c+1}} = \frac{dl_{dD_{c,c+1}} Q^{-1}_{dD_{c,c+1}} dl_{dD_{c,c+1}}}{n_D \cdot s^2}, c = 1, \ldots, (Sim - 1), \tag{6}$$

where

$dl_{dD_{c,c+1}} = D_{ij_{c+1}} - D_{ij_c}$ (7) is the change of the distance between two points P_i and P_j in two simulations,

$$D_{ij_c} = \sqrt{\left(\hat{y}_{j_c} - \hat{y}_{i_c}\right)^2 + \left(\hat{x}_{j_c} - \hat{x}_{i_c}\right)^2} \text{ and } D_{ij_{c+1}} = \sqrt{\left(\hat{y}_{j_{c+1}} - \hat{y}_{i_{c+1}}\right)^2 + \left(\hat{x}_{j_{c+1}} - \hat{x}_{i_{c+1}}\right)^2} \tag{8}$$

are the distances between the points P_i and P_j in the c^{th} and $(c + 1)^{\text{th}}$ simulation, $\hat{y}_{i_c}$, $\hat{x}_{i_c}$ and $\hat{y}_{j_c}$, $\hat{x}_{j_c}$ are adjusted coordinates of the points P_i and P_j written in the vector $\hat{\mathbf{x}}_c$ in the c^{th} simulation and $\hat{y}_{i_{c+1}}$, $\hat{x}_{i_{c+1}}$ and $\hat{y}_{j_{c+1}}$, $\hat{x}_{j_{c+1}}$ are the adjusted coordinates of the points P_i and P_j, written in the vector $\hat{\mathbf{x}}_{c+1}$ in the $(c + 1)^{\text{th}}$ simulation. The element of the cofactor matrix of the distance differences $Q_{dD_{c,c+1}}$ in Equation (6) are computed by:

$$Q_{dD_{c,c+1}} = \mathbf{L}_{dD_{ij}} \mathbf{Q}_{\mathbf{u}dD_{ij}} \mathbf{L}^{\mathrm{T}}_{dD_{ij}}, \tag{9}$$

where the vector of the partial derivatives of the measurements is

$$\mathbf{L}_{dD_{ij}} = [-\sin \nu_{ij},\ \ -\cos \nu_{ij},\ \ \sin \nu_{ij},\ \ \cos \nu_{ij}] \text{ and} \tag{10}$$

$$\nu_{ij} = (\nu_{ij_c} + \nu_{ij_{c+1}})/2 \tag{11}$$

is the average value of the bearing angles between the points P_i and P_j between two simulations,

$$\nu_{ij_c} = \arctan \frac{\hat{y}_{j_c} - \hat{y}_{i_c}}{\hat{x}_{j_c} - \hat{x}_{i_c}} \text{ and } \nu_{ij_{c+1}} = \arctan \frac{\hat{y}_{j_{c+1}} - \hat{y}_{i_{c+1}}}{\hat{x}_{j_{c+1}} - \hat{x}_{i_{c+1}}} \tag{12}$$

are bearing angles between the points P_i and P_j in the c^{th} and $(c + 1)^{\text{th}}$ simulation. In submatrix $\mathbf{Q}_{\mathbf{u}dD_{ij}}$ are the corresponding elements of the matrix $\mathbf{Q}_{\mathbf{u}}$, which refer to the points P_i and P_j.

In order to confirm the correct number of degrees of freedom $n_D = 1$ when computing the test statistics (6) and the critical values for determine the limits of the reject region, the computed test statistics $T^2_{2D_{c,c+1}}$ should be sorted according to the values from smallest to largest according to Equation (6) and plotted on the empirical cumulative distribution function (CDF) graph.

3.3 Testing the changes of angles in geodetic network

When testing angular changes in the geodetic network, which is presented as a novelty in the article presented by (Savšek and Ambrožič, 2023) on testing the transformation of the geodetic network, test statistics between two simulations $T^2_{2\alpha_{c,c+1}}$ according to (Savšek and Ambrožič, 2023) is compiled:

$$T^2_{2\alpha_{c,c+1}} = \frac{dl_{d\alpha_{c,c+1}} Q^{-1}_{d\alpha_{c,c+1}} dl_{d\alpha_{c,c+1}}}{n_\alpha \cdot s^2}, c = 1, \ldots, (Sim - 1), \tag{13}$$

where

$$dl_{d\alpha_{c,c+1}} = \alpha_{ijk_{c+1}} - \alpha_{ijk_c} \tag{14}$$

is the change in the angle between the points P_i, P_j and P_k between two simulations,

$$\alpha_{ijk_c} = \nu_{ik_c} - \nu_{ij_c} \text{ and } \alpha_{ijk_{c+1}} = \nu_{ik_{c+1}} - \nu_{ij_{c+1}} \tag{15}$$

are angles in P_i between P_j and P_k in the c^{th} and $(c + 1)^{\text{th}}$ simulation, ν_{ij_c}, ν_{ik_c}, $\nu_{ij_{c+1}}$ and $\nu_{ik_{c+1}}$ are bearing angles from adjusted coordinates between the points P_i and P_j and between P_i and P_k in the c^{th} and

$(c+1)^{\text{th}}$ simulation. The element of the cofactor matrix of angular changes $Q_{d\alpha_{c,c+1}}$ in Equation (13) are computed by

$$Q_{d\alpha_{c,c+1}} = \mathbf{L}_{d\alpha_{ijk}} \mathbf{Q}_{\mathbf{u}d\alpha_{ijk}} \mathbf{L}^{\mathrm{T}}_{d\alpha_{ijk}}, \tag{16}$$

where the vector of the partial derivatives of the measurements is

$$\mathbf{L}_{d\alpha_{ijk}} = \left[\left(-\frac{\cos\nu_{ik}}{D_{ik}} + \frac{\cos\nu_{ij}}{D_{ij}}\right), \left(\frac{\sin\nu_{ik}}{D_{ik}} - \frac{\sin\nu_{ij}}{D_{ij}}\right), \left(-\frac{\cos\nu_{ij}}{D_{ij}}\right), \left(\frac{\sin\nu_{ij}}{D_{ij}}\right), \left(\frac{\cos\nu_{ik}}{D_{ik}}\right), \left(-\frac{\sin\nu_{ik}}{D_{ik}}\right)\right], \tag{17}$$

$$\nu_{ij} = (\nu_{ij_c} + \nu_{ij_{c+1}})/2 \text{ and } \nu_{ik} = (\nu_{ik_c} + \nu_{ik_{c+1}})/2 \tag{18}$$

are average values of bearing angles,

$$D_{ij} = (D_{ij_c} + D_{ij_{c+1}})/2 \text{ and } D_{ik} = (D_{ik_c} + D_{ik_{c+1}})/2 \tag{19}$$

are average values of the distances from adjusted coordinates between the points P_i and P_j and between P_i and P_k between two simulations. In the submatrix $\mathbf{Q}_{\mathbf{u}d\alpha_{ijk}}$ are the corresponding elements of a matrix $\mathbf{Q}_{\mathbf{u}}$ which refer to the points P_i, P_j and P_k.

To confirm the correct number of degrees of freedom $n_\alpha = 1$ when computing the test statistics (13) and the critical values for determine the limits of the reject region, the computed test statistics $T^2_{2\alpha_{c,c+1}}$ according to Equation (13) should be sorted by the values from smallest to largest and plotted on the empirical CDF graph.

3.4 Testing the changes coordinates of the triangle vertices in the geodetic network

When testing changes in the coordinates of the vertices of triangles in the geodetic network, which is one of the components of testing the transformation of the geodetic network, test statistics are generated between two simulations $T^2_{2\Delta_{c,c+1}}$ (Welsch, 1983; Welsch and Zhang, 1983) and according to (Savšek and Ambrožič, 2023):

$$T^2_{2\Delta_{c,c+1}} = \frac{\mathbf{u}^{\mathrm{T}}_{\Delta_{c,c+1}} \mathbf{Q}^{-1}_{\mathbf{u}\Delta} \mathbf{u}_{\Delta_{c,c+1}}}{n_\Delta \cdot s^2}, c = 1,\ldots,(Sim-1), \tag{20}$$

where

$$\mathbf{u}_{\Delta_{c,c+1}} = \hat{\mathbf{x}}_{c+1} - \hat{\mathbf{x}}_c \tag{21}$$

is the vector of displacements or coordinate changes of the triangle vertices P_i, P_j and P_k between two simulations. The submatrix $\mathbf{Q}_{\mathbf{u}\Delta}$ contains the corresponding elements of the cofactor matrix $\mathbf{Q}_{\mathbf{u}}$, which refer to the points P_i, P_j and P_k.

In order to confirm that the number of degrees of freedom n_Δ is equal to 3 when computing the test statistics (20) and the critical values to determine the limits of the reject region, the computed test statistics $T^2_{2\Delta_{c,c+1}}$ according to Equation (20) should be sorted by values from smallest to largest and plotted on the empirical CDF graph.

```
Start
  • Read input data:
      • adjusted coordinates of points $\hat{\mathbf{x}}_c$, $\hat{\mathbf{x}}_{c+1}$
      • cofactor matrices of coordinate unknowns $\mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c}$, $\mathbf{Q}_{\hat{\mathbf{x}}_{c+1}\hat{\mathbf{x}}_{c+1}}$
      • reference variances a posteriori $s_c^2$, $s_{c+1}^2$
      • number of redundant measurements $f_c$, $f_{c+1}$
  • For all simulations $c = 1, ..., Sim\text{-}1$:
      • Testing the congruence of geodetic network:
          • $\mathbf{u}_{c,c+1} = \hat{\mathbf{x}}_{c+1} - \hat{\mathbf{x}}_c$
          • $\mathbf{Q}_\mathbf{u} = \mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c} + \mathbf{Q}_{\hat{\mathbf{x}}_{c+1}\hat{\mathbf{x}}_{c+1}} = 2\mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c}$
          • $f = f_c + f_{c+1}, f_\mathbf{u} = \text{rank } \mathbf{Q}_\mathbf{u} = u - d$
          • $s^2 = \frac{f_c s_c^2 + f_{c+1} s_{c+1}^2}{f_c + f_{c+1}} \ldots (4)$
          • $T^2_{1_{c,c+1}} = \frac{s_\mathbf{u}^2}{s^2} = \frac{\mathbf{u}^\mathrm{T}_{c,c+1} \mathbf{Q}_\mathbf{u}^{-1} \mathbf{u}_{c,c+1}}{f_\mathbf{u} \cdot s^2} \ldots (5)$
          • $F_{f_\mathbf{u},f} = \text{fcdf}(f_\mathbf{u}, f)$
      • For all distances – testing the changes of distances in the geodetic network:
          • $dl_{dD_{c,c+1}} = D_{ij_{c+1}} - D_{ij_c} \ldots (7)$
          • $\nu_{ij} = (\nu_{ij_c} + \nu_{ij_{c+1}})/2 \ldots (11)$
          • $\mathbf{L}_{dD_{ij}} = [-\sin\nu_{ij}, \ -\cos\nu_{ij}, \ \sin\nu_{ij}, \ \cos\nu_{ij}] \ldots (10)$
          • $Q_{dD_{c,c+1}} = \mathbf{L}_{dD_{ij}} \mathbf{Q}_{\mathbf{u}dD_{ij}} \mathbf{L}^\mathrm{T}_{dD_{ij}} \ldots (9)$
          • $n_D = 1$
          • $T^2_{2D_{c,c+1}} = \frac{dl_{dD_{c,c+1}} Q^{-1}_{dD_{c,c+1}} dl_{dD_{c,c+1}}}{n_D \cdot s^2} \ldots (6)$
          • $F_{n_D,f} = \text{fcdf}(n_D, f) = \text{fcdf}(1, f)$
      • End of testing the changes of distances in the geodetic network
      • For all angles – testing the changes of angles in the geodetic network:
          • $\alpha_{ijk_c} = \nu_{ik_c} - \nu_{ij_c}, \alpha_{ijk_{c+1}} = \nu_{ik_{c+1}} - \nu_{ij_{c+1}} \ldots (15)$
          • $dl_{d\alpha_{c,c+1}} = \alpha_{ijk_{c+1}} - \alpha_{ijk_c} \ldots (14)$
          • $\nu_{ij} = (\nu_{ij_c} + \nu_{ij_{c+1}})/2, \nu_{ik} = (\nu_{ik_c} + \nu_{ik_{c+1}})/2 \ldots (18)$
          • $D_{ij} = (D_{ij_c} + D_{ij_{c+1}})/2, D_{ik} = (D_{ik_c} + D_{ik_{c+1}})/2 \ldots (19)$
          • $\mathbf{L}_{d\alpha_{ijk}} = \left[\left(-\frac{\cos\nu_{ik}}{D_{ik}} + \frac{\cos\nu_{ij}}{D_{ij}}\right), \left(\frac{\sin\nu_{ik}}{D_{ik}} - \frac{\sin\nu_{ij}}{D_{ij}}\right), \left(-\frac{\cos\nu_{ij}}{D_{ij}}\right), \left(\frac{\sin\nu_{ij}}{D_{ij}}\right), \left(\frac{\cos\nu_{ik}}{D_{ik}}\right), \left(-\frac{\sin\nu_{ik}}{D_{ik}}\right)\right] \ldots (17)$
          • $Q_{d\alpha_{c,c+1}} = \mathbf{L}_{d\alpha_{ijk}} \mathbf{Q}_{\mathbf{u}d\alpha_{ijk}} \mathbf{L}^\mathrm{T}_{d\alpha_{ijk}} \ldots (16)$
          • $n_\alpha = 3$
          • $T^2_{2\alpha_{c,c+1}} = \frac{dl_{d\alpha_{c,c+1}} Q^{-1}_{d\alpha_{c,c+1}} dl_{d\alpha_{c,c+1}}}{n_\alpha \cdot s^2} \ldots (13)$
          • $F_{n_\alpha,f} = \text{fcdf}(n_\alpha, f) = \text{fcdf}(1, f)$
      • End of testing the changes of angles in the geodetic network
      • For all triangles – testing the changes coordinates of the triangle vertex in the geodetic network:
          • $\mathbf{u}_{\Delta_{c,c+1}} = \hat{\mathbf{x}}_{c+1} - \hat{\mathbf{x}}_c \ldots (21)$
          • $n_\Delta = 1$
          • $T^2_{2\Delta_{c,c+1}} = \frac{\mathbf{u}^\mathrm{T}_{\Delta_{c,c+1}} \mathbf{Q}^{-1}_{\mathbf{u}\Delta} \mathbf{u}_{\Delta_{c,c+1}}}{n_\Delta \cdot s^2} \ldots (20)$
          • $F_{n_\Delta,f} = \text{fcdf}(n_\Delta, f) = \text{fcdf}(3, f)$
      • End of testing the changes coordinates of the triangle vertex in the geodetic network
      • Plot distribution functions
  • End of simulations
End
```

Figure 2: Flow chart of the simulations for determination of the number of degrees of freedom.

4 Numerical example

To confirm the correct number of degrees of freedom in the computation of limits of the reject region an example of a 2D geodetic network is considered. It is used to demonstrate the functioning of all procedures of deformation analysis methods and investigated by many researchers (Figure 3). The case of the considered 2D geodetic network has been studied also in our research group in several scientific articles (Ambrožič, 2001, 2004; Ambrožič et al., 2019; Hamza et al., 2020; Marjetič et al., 2012; Savšek and Ambrožič, 2023; Soldo and Ambrožič, 2018; Vrečko and Ambrožič, 2013) and (Batilović et al., 2022). The correctly selected number of degrees of freedom is shown by means of examples for the computation of critical values when testing congruence, the changes of distances, angles and coordinates of triangle vertices in the geodetic network.

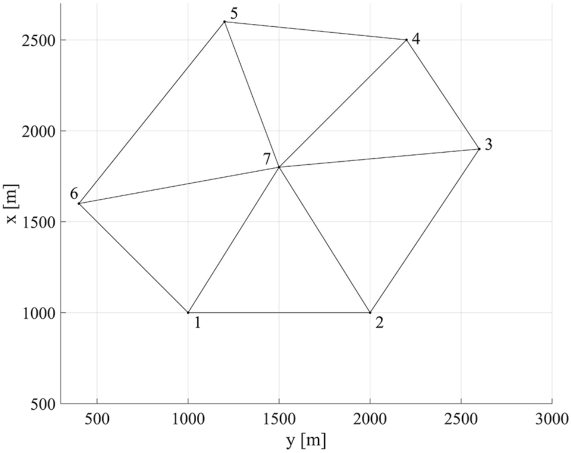

Figure 3: Geodetic network.

4.1 Number of degrees of freedom when testing the congruence

When testing the congruence of the geodetic network, the test statistics $T^2_{1_{c,c+1}}$, $c = 1, \ldots, (Sim - 1)$ are computed according to Equation (5). They are sorted according to their values from smallest to largest. The graph with the primary axis corresponding to $T^2_{1_{c,c+1}}$, and the secondary axis corresponding to $c/(Sim - 1)$, $c = 1, \ldots, (Sim - 1)$ is plotted. Figure 4 shows the graphs of the test statistics for different values of the first parameter of degrees of freedom $f_\mathbf{u}$ and the Cumulative Distribution Function (CDF) of the F-distribution at the correct number of degrees of freedom $f_\mathbf{u}$ and f.

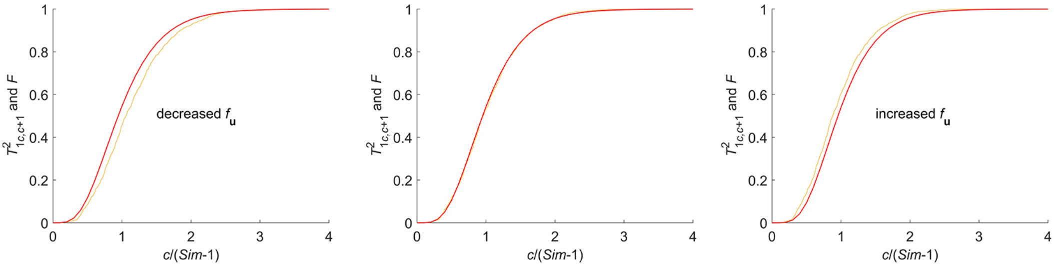

Figure 4: Graphs of the probability distribution function of the test statistic when testing the congruence of geodetic network (smooth thin brown line) for $Sim = 1000$ simulations and the cumulative distribution function of the F-distribution (smooth thickened red line).

Figure 4 clearly shows that the graph of the probability distribution of the test statistics (5) perfectly matches the graph of the cumulative distribution function of the F-distribution for the first parameter of degrees of freedom $f_\mathbf{u} = u - d$ (Figure 4 middle), which is not the case when the first number of $f_\mathbf{u}$ is decreased ($f_\mathbf{u} = u - d - 1$; Figure 4 left) or increased by one ($f_\mathbf{u} = u - d + 1$; Figure 4 right).

Figure 5 shows graphs of the test statistics for different values of the second parameter of degrees of freedom f and the cumulative distribution function of the F-distribution at the correct number of degrees of freedom $f_\mathbf{u}$ and f.

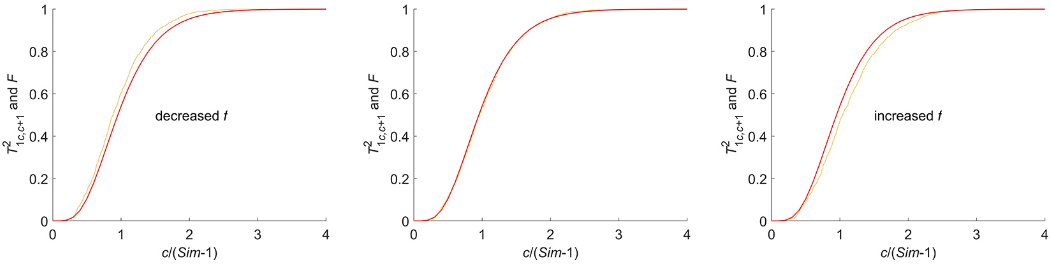

Figure 5: Graphs of the probability distribution function of the test statistic when testing the congruence of geodetic network (smooth thin brown line) for *Sim* = 1000 simulations and the cumulative distribution function of the F-distribution (smooth thickened red line).

Figure 5 clearly shows that the graph of the probability distribution of the test statistic (5) perfectly matches the graph of the cumulative distribution function of F-distribution for the second parameter of degrees of freedom $f = f_1 + f_2$ (Figure 5 middle), which is not the case if the second number of degrees of freedom is decreased ($f = f_1 + f_2 - 5$; Figure 5 left) or increased by five ($f = f_1 + f_2 + 5$; Figure 5 right). If the number of degrees of freedom is increased or decreased by one, the difference would be less visible.

4.2 Number of degrees of freedom when testing the changes of distances

When testing distance changes in the geodetic network, the test statistics $T^2_{2D_{c,c+1}}$, $c = 1, \ldots, (Sim - 1)$, are computed according to Equation (6). They are sorted by their values from smallest to largest. The graph with the primary axis corresponding to $T^2_{2D_{c,c+1}}$, and the secondary axis corresponding to $c/(Sim - 1)$, $c = 1, \ldots, (Sim - 1)$ is plotted. Figure 6 shows graphs with different numbers of simulations of the test statistics for all 21 distances between points at the same number of degrees of freedom ($n_D = 1$).

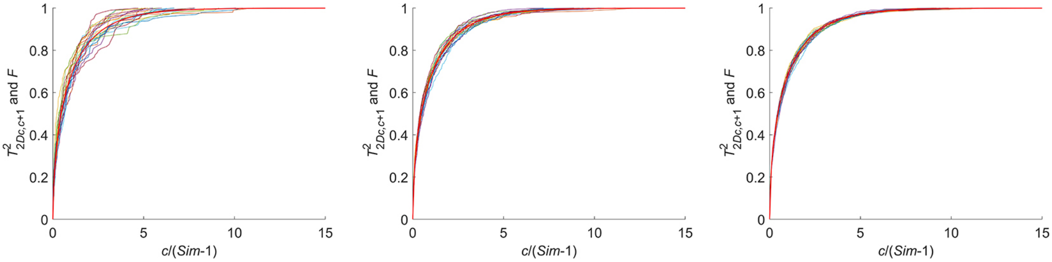

Figure 6: Graphs of probability distribution function of test statistic (6) at different numbers of simulations.

Figure 6 shows that the probability distribution graphs already take the correct shape at 100 simulations (Figure 6 left), at 500 simulations the distribution graphs are quite similar regardless of which distance is considered (Figure 6 middle), and at 1000 simulations we no longer distinguish which distance the graphs refer to (Figure 6 right).

To confirm the correct number of degrees of freedom, the distribution of the test statistic (6) is shown in Figure 7 for all 21 distances and the cumulative distribution function of F-distribution at different degrees of freedom.

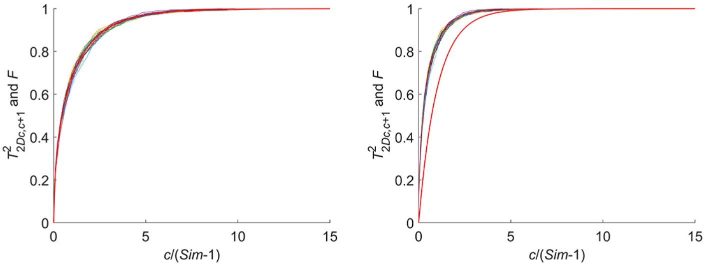

Figure 7: Graphs of the probability distribution function of the test statistic for changes in distances (lines of different colours) for *Sim* = 1000 simulations and different numbers of the degrees of freedom and the cumulative distribution function of F-distribution (smooth thickened red line).

Figure 7 clearly shows that the graphs of distribution of the test statistics (6) exactly match the graph of the cumulative distribution function of F-distribution at number of degrees of freedom $n_D = 1$ (Figure 7 left), which is not the case when the number of the degrees of freedom is $n_D = 2$ (Figure 7 right).

4.3 Number of degrees of freedom when testing the angular changes

When testing angular changes in the geodetic network, the test statistics $T^2_{2\alpha_{c,c+1}}$, $c = 1, \ldots, (Sim - 1)$, are computed according to Equation (13). They are sorted by their values from smallest to largest. The graph with the primary axis corresponding to $T^2_{2\alpha_{c,c+1}}$, and the secondary axis corresponding to $c/(Sim - 1)$, $c = 1, \ldots, (Sim - 1)$ is plotted.

To confirm the correct number of degrees of freedom, in Figure 8 the distribution of the test statistic (13) is plotted for the first 105 angles and the cumulative distribution function of the F-distribution at different degrees of freedom.

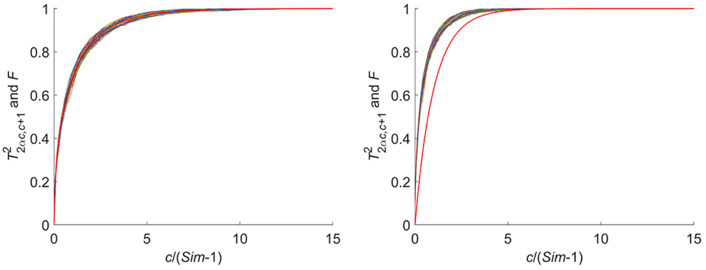

Figure 8: Graphs of the probability distribution function of the test statistic for angular changes (lines of different colours) for *Sim* = 1000 simulations and different numbers of the degrees of freedom and the cumulative distribution function of the F-distribution (smooth thickened red line).

Figure 8 clearly shows that the graphs of the probability distribution of the test statistics (13) exactly match the graph of the cumulative distribution function of F-distribution at the number of degrees of freedom $n_\alpha = 1$ (Figure 8 left), which is not the case when the number of degrees of freedom is $n_\alpha = 2$ (Figure 8 right).

4.4 Number of degrees of freedom when testing changes in the coordinates of the triangle vertices

When testing changes in the coordinates of the vertices of triangles in the geodetic network, the test statistic $T^2_{2\Delta_{c,c+1}}$, $c = 1, \ldots, (Sim - 1)$ are computed according to Equation (20). They are sorted according to their values from smallest to largest. The graph with the primary axis corresponding to $T^2_{2\Delta_{c,c+1}}$, and the secondary axis corresponding to $c/(Sim - 1)$, $c = 1, \ldots, (Sim - 1)$ is plotted.

To confirm the correct number of degrees of freedom, Figure 9 shows the distribution of the test statistics (20) for all 35 triangles and the cumulative distribution function of the F-distribution at different numbers of degrees of freedom.

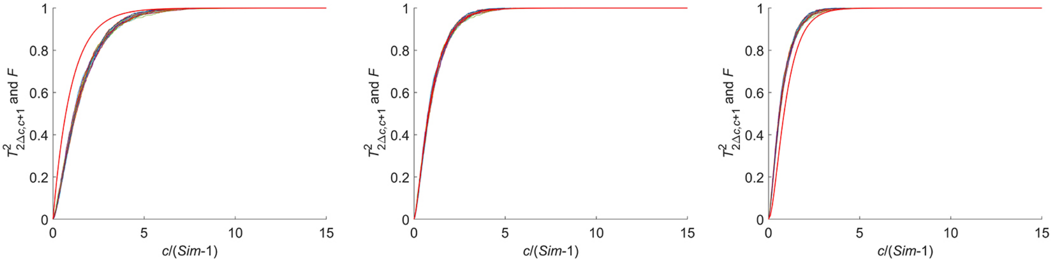

Figure 9: Graphs of the probability distribution function of the test statistic for changes in the coordinates of the triangle vertices (lines of different colours) for *Sim* = 1000 simulations and different numbers of the degrees of freedom and the cumulative distribution function of the F-distribution (smooth thickened red line).

Figure 9 clearly shows that the graphs of the probability distribution of the test statistics (20) match perfectly with the graph of the cumulative distribution function of the F-distribution at the number of degrees of freedom $n_\Delta = 3$ (Figure 9 middle), which is not the case when the number of the degrees of freedom is $n_\Delta = 2$ (Figure 9 left) or $n_\Delta = 4$ (Figure 9 right).

The experiments in Figures 4-9 confirms that the correct number of degrees of freedom can also be computed using the Monte Carlo method.

5 Discussion and conclusions

In this study, another way of applying the Monte Carlo method is shown, namely to confirm the correct number of degrees of freedom in the computation of the critical values (limits of the reject region). The measurements of horizontal directions and horizontal distances were simulated in a 2D geodetic network and adjusted using the Least Squares Method. The simulations of measurements and network adjustments were repeated 1000 times. The value of 1000 simulations were chosen because at this value the graphs of the probability distribution function of the test statistic almost perfectly coincided with the graph of the cumulative distribution function of the F-distribution. Between two consecutive network adjustments, representing two time measurements, the test statistics used to test the congruence, changes in distances, angles and coordinates of the vertices of the triangles in the geodetic network were computed. The test statistics for each tested variable were sorted according to their value and presented in the graph together with the cumulative distribution function of the F-distribution, according to which the test statistics are distributed.

The aim of the study was to empirically confirm the probability distribution and the number of degrees of freedom for testing the congruence of geodetic network, the changes in distances, angles and coordinates of triangle vertices as proposed in the Munich approach of deformation analysis. With results we can confirm that:

1. The test statistic in testing the congruence of geodetic network between two measurements is distributed according to F-distribution $F_{f_{\mathbf{u}},f}$, i.e. with the degrees of freedom $f_{\mathbf{u}}$ and f,
2. The test statistic for the change of distance between two points P_i and P_j is distributed in two measurements according to the F-distribution $F_{1,f}$, i.e. with the first degree of freedom $n_D = 1$,
3. The test statistic for the change of coordinate of vertices in triangle P_i, P_j and P_k in two measurements is distributed according to the F-distribution $F_{3,f}$, i.e. with the first degree of freedom $n_\Delta = 3$.

In addition, it was found out that:

4. The test statistic for the change of angles between points P_i, P_j and P_k in two measurements is distributed according to the F-distribution $F_{1,f}$, i.e. with the first degree of freedom $n_\alpha = 1$.

The values of degrees of freedom cannot be found in any literature and is considered as the most important contribution of this work. The appropriate number of degrees of freedom is important for determine the limits of reject region when testing various test statistics in the Munich approach and is therefore crucial for choosing the right decisions in the process of analyzing the measured deformations of different objects.

Acknowledgements

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Testiranje uporabe metode simulacije Monte Carlo za določitev števila prostostnih stopenj v deformacijski analizi postopka München

OSNOVNE INFORMACIJE O ČLANKU

GLEJ STRAN 187

1 Uvod

Deformacijska analiza po postopku München je geodetsko-statistična metoda, ki kot splošno lastnost vseh deformacijskih analiz določa spremljanje premikov kontrolnih točk na opazovanem objektu glede na izbrane referenčne točke na okoliškem terenu. Pri različnih metodah deformacijske analize: postopek Hannover (Pelzer, 1971), postopek Delft (Heck et al., 1982), postopek Karlsruhe (Heck, 1983), postopek Fredericton (Chrzanowski, 1981) in postopek München (Welsch, 1982, 1983; Welsch in Zhang, 1983) poskušamo s statističnim testiranjem zavrniti ničelno domnevo o ničelnih premikih točk v geodetskih mrežah.

Postopek München so razvili v Münchnu v Nemčiji in je bil obravnavan v različnih geodetskih publikacijah (Welsch, 1982, 1983; Welsch in Zhang, 1983). Je del širšega področja deformacijske analize, ki se uporablja v geodeziji in geodinamiki za preučevanje deformacij naravnih in umetnih objektov, deformacij skorje in deformacij težnostnega polja. Postopek temelji na statistični in geometrijski analizi deformacijskih merjenj in je pomembno orodje v inženirski geodeziji (Caspary et al., 1990; Dermanis in Livieratos, 1983; Nowel, 2015).

V statistiki je preizkušanje domnev ključnega pomena za sprejemanje odločitev na podlagi vzorčnih podatkov. Pomaga nam pri nadzoru pogreškov, preverjanju znanstvene veljavnosti in sklepanju o populacijah. V naši raziskavi smo metodo uporabili v deformacijski analizi pri raziskovanju deformiranega stanja naravnih ali umetnih objektov. Testiranje domnev na splošno izvajamo po naslednjem postopku (Lehmann in Romano, 2022):

- postavimo ničelno H_0 in alternativno domnevo H_α o določenih lastnostih populacije,
- izberemo testno statistiko, ki ustreza ničelni domnevi in za katero poznamo porazdelitev verjetnosti,
- izberemo tveganje oziroma stopnjo značilnosti α in na njeni osnovi ter na podlagi porazdelitve testne statistike določimo meje območja zavrnitve ničelne domneve,
- na vzorčnih podatkih izračunamo vrednost testne statistike,
- oblikujemo sklep:
  - če vrednost testne statistike leži v območju zavrnitve ničelne domneve, potem ničelno domnevo zavrnemo ob tveganju α,
  - če vrednost testne statistike ne leži v območju zavrnitve ničelne domneve, potem ničelne domneve ne moremo zavrniti ob tveganju α.

Pri določanju porazdelitve izbrane testne statistike izračunamo meje območja zavrnitve ničelne domneve iz inverzne kumulativne porazdelitvene funkcije. V nekaterih primerih je porazdelitev testne statistike Fisherjeva porazdelitev verjetnosti (porazdelitev F). Da bi lahko pravilno izračunali meje območja zavrnitve ničelne domneve, moramo poznati pravilno število prostostnih stopenj.

Premike nestabilnih točk je mogoče identificirati pri tveganju le, če je natančnost določitve premikov ustrezna. V tem kontekstu je pomembna mejna vrednost, ki predstavlja spodnjo mejo premika nestabilnih točk, tj. minimalno zaznavno deformacijo.

V deformacijski analizi po postopku München (Savšek in Ambrožič, 2023; Welsch, 1983; Welsch in Zhang, 1983) se:

– testna statistika za razlike koordinat vseh točk v dvodimenzionalni geodetski mreži (testiranje skladnosti) med dvema izmerama porazdeljuje po porazdelitvi F ($F_{f_\mathbf{u},f}$),
– testna statistika za spremembo dolžine med točkama P_i in P_j med dvema izmerama porazdeljuje po porazdelitvi F ($F_{1,f}$),
– testna statistika za spremembo kota med točkami P_i, P_j in P_k med dvema izmerama porazdeljuje po porazdelitvi F ($F_{1,f}$),
– testna statistika za spremembo koordinat oglišč v trikotniku P_i, P_j in P_k med dvema izmerama porazdeljuje po porazdelitvi F ($F_{3,f}$),

kjer je $f_\mathbf{u} = u - d$ število prostostnih stopenj, $u = 2m$ je število koordinatnih neznank v 2D mreži (m je število vseh točk v geodetski mreži), d je defekt datuma geodetske mreže, $f = f_1 + f_2$ je število nadštevilnih meritev v izravnavi predhodne f_1 in tekoče f_2 terminske izmere. Za pravilen izračun kritičnih vrednosti $F_{f_\mathbf{u},f}$, $F_{1,f}$ in $F_{3,f}$ torej potrebujemo pravilno število prostostnih stopenj.

Za potrditev pravilnega števila prostostnih stopenj v izračunu mej območja zavrnitve ničelne domneve lahko uporabimo simulacijo Monte Carlo, kar je tudi cilj te raziskave. V raziskavi se večinoma opiramo na naše predhodne raziskave, ki sta jih izvedla Savšek in Ambrožič (Savšek in Ambrožič, 2023), kar je logična nadgradnja prejšnjih raziskav.

2 Simulacija meritev in izravnava

Raziskavo začnemo s simulacijo meritev v geodetski mreži, torej horizontalnih smeri in dolžin s programom, napisanim v ta namen. Predpostavimo, da meritve vsebujejo le slučajne pogreške, grobih in sistematičnih pogreškov ne smejo vsebovati. Zato lahko predpostavimo, da so meritve porazdeljene normalno okoli srednje vrednosti $\overline{y}_{meas}$ s standardno deviacijo σ. Če torej želimo simulirati meritve, moramo pričakovanim vrednostim meritev prišteti normalno porazdeljen slučajni pogrešek $u_k\sigma$:

$$y_k = \overline{y}_{meas} + u_k\sigma, \tag{1}$$

kjer je u_k standardno normalno porazdeljena slučajna spremenljivka.

Pričakovane vrednosti meritev izračunamo iz koordinat točk, ki jih seveda poznamo, saj želimo simulirati meritve med temi točkami (Kuang, 1996).

Simulirano k-to horizontalno smer s_k (oziroma smerni kot, saj pri simulacijah orientacija smeri na stojišču ni pomembna) v triangulacijski geodetski mreži dobimo z naslednjo enačbo:

$$s_k = \overline{s}_{koo} + u_{1k}\sigma_{hz}, \tag{2}$$

kjer je $\overline{s}_{koo} = \arctan\dfrac{y_j - y_i}{x_j - x_i}$ smerni kot med točkama P_i in P_j, σ_{hz} je standardna deviacija izbrane smeri in u_{1k} standardno normalno porazdeljena slučajna spremenljivka.

Simulirano k-to horizontalno dolžino d_k v trilateracijski geodetski mreži dobimo z naslednjo enačbo:

$$d_k = \bar{d}_{koo} + u_{2k}\sigma_d, \tag{3}$$

kjer je $\bar{d}_{koo} = \sqrt{\left(y_j - y_i\right)^2 + \left(x_j - x_i\right)^2}$ horizontalna dolžina med točkama P_i in P_j, σ_d je standardna deviacija izbrane horizontalne dolžine in u_{2k} standardno normalno porazdeljena slučajna spremenljivka.

V naslednjem koraku te simulirane meritve izravnamo z metodo najmanjših kvadratov, kot bi izravnali dejansko opravljene meritve, in sicer kot vpeto geodetsko mrežo, kjer velja $\mathbf{v}^\mathrm{T}\mathbf{P}\mathbf{v}$ = min., ali kot prosto mrežo, kjer veljata pogoja $\mathbf{v}^\mathrm{T}\mathbf{P}\mathbf{v}$ = min. in $\hat{\mathbf{x}}^\mathrm{T}\hat{\mathbf{x}}$ = min. (Kuang, 1996). Tako dobimo izravnane koordinate novih točk v mreži in referenčno varianco a posteriori s^2.

```
Začetek
  • Čitanje vhodnih podatkov:
      • približne koordinate točk $\mathbf{x}$
      • standardna deviacija smeri $\sigma_{hz}$
      • standardna deviacija dolžin $\sigma_d$
  • Za vse simulacije $c = 1, ..., Sim$:
      • za vse vizure v geodetski mreži $k = 1, ..., n$:
          • generiranje vzorca standardizirano normalno
            porazdeljenih slučajnih spremenljivk:
              • $u_{1k}$ = randn, $u_{2k}$ = randn
          • izračun meritev iz približnih koordinat točk $P_i$ in $P_j$:
              • $\bar{s}_{koo} = \arctan \frac{y_j - y_i}{x_j - x_i}$
              • $\bar{d}_{koo} = \sqrt{(y_j - y_i)^2 + (x_j - x_i)^2}$
          • izračun simuliranih meritev:
              • $s_k = \bar{s}_{koo} + u_{1k}\sigma_{hz}$ … (2)
              • $d_k = \bar{d}_{koo} + u_{2k}\sigma_d$ … (3)
      • Konec vizur v geodetski mreži
      • Izravnava simuliranih meritev v geodetski mreži:
          • vpeta mreža:                     • prosta mreža:
            $\mathbf{v}^T\mathbf{P}\mathbf{v}$ = min.       $\mathbf{v}^T\mathbf{P}\mathbf{v}$ = min. in $\hat{\mathbf{x}}^T\hat{\mathbf{x}}$ = min.
      • Rezultati izravnave:
          • izravnane koordinate točk $\hat{\mathbf{x}}_c$
          • matrika kofaktorjev koordinatnih neznank $\mathbf{Q}_{\hat{x}_c\hat{x}_c}$
          • referenčna varianca a posteriori $s_c^2$
          • število nadštevilnih meritev $f_c$
  • Konec simulacij
Konec
```

Slika 1: Diagram poteka simulacij meritev in izravnave.

Po opisanem postopku simulacije meritev in izravnave dobimo rezultate prve iteracije oziroma simulacije. Postopek simulacije meritev in izravnave moramo ponoviti *Sim*-krat, kjer je *Sim* število simulacij. Rezultat tega dela raziskave so torej izravnane koordinate točk geodetske mreže $\hat{\mathbf{x}}_c$ in referenčne variance a posteriori s_c^2, kjer je $c = 1, \ldots, Sim$. Ker geometrije in natančnosti meritev σ_{hz} and σ_d v simulacijah meritev ne spreminjamo, je pripadajoča matrika kofaktorjev koordinatnih neznank $\mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c}$ enaka v vseh simulacijah. Tako v vseh iteracijah ostane enaka tudi matrika kofaktorjev koordinatnih razlik $\mathbf{Q}_\mathbf{u} = \mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c} + \mathbf{Q}_{\hat{\mathbf{x}}_{c+1}\hat{\mathbf{x}}_{c+1}} = 2\mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c}$, $c = 1, \ldots, (Sim - 1)$ in jo izračunamo, kot je opisano v Savšek in Ambrožič (2023).

3 Testiranje preoblikovanja geodetske mreže

Po izvedenih simulacijah imamo izpolnjena pogoja o homogenosti natančnosti meritev in identičnih točkah, saj simulacije ponavljamo na isti geodetski mreži. Zato lahko med dvema simulacijama, ki predstavljata dve terminski izmeri, izračunamo referenčno varianco a posteriori s^2 po enačbi iz Savšek in Ambrožič (2023):

$$s^2 = \frac{f_c s_c^2 + f_{c+1} s_{c+1}^2}{f_c + f_{c+1}}, c = 1, \ldots, (Sim - 1), \tag{4}$$

kjer sta f_c in f_{c+1} števili nadštevilnih meritev v dveh zaporednih simulacijah, s_c^2 in s_{c+1}^2 sta referenčni varianci a posteriori v dveh zaporednih simulacijah.

3.1 Testiranje skladnosti geodetske mreže

Testiranje skladnosti geodetske mreže, kar je naslednja faza postopka München, ne pomeni težave, saj sestavimo testno statistiko med dvema zaporednima simulacijama (Welsch, 1983; Welsch in Zhang, 1983; Kuang, 1996):

$$T_{1_{c,c+1}}^2 = \frac{s_\mathbf{u}^2}{s^2} = \frac{\mathbf{u}_{c,c+1}^\mathbf{T} \mathbf{Q}_\mathbf{u}^{-1} \mathbf{u}_{c,c+1}}{f_\mathbf{u} \cdot s^2}, c = 1, \ldots, (Sim - 1), \tag{5}$$

kjer je $\mathbf{u}_{c,c+1}$ vektor premikov (razlik koordinat) točk med dvema zaporednima simulacijama, $\mathbf{Q}_\mathbf{u}$ je matrika kofaktorjev koordinatnih razlik. Izračun kritične vrednosti $F_{f_\mathbf{u}, f}$ tudi ni težaven, saj sta prostostni stopnji $f_\mathbf{u}$ in f znani.

3.2 Testiranje sprememb dolžin v geodetski mreži

Pri testiranju sprememb dolžin med točkami v geodetski mreži, ki je eden izmed sestavnih delov testiranja preoblikovanja geodetske mreže po postopku München, sestavimo testno statistiko med dvema zaporednima simulacijama $T_{2D_{c,c+1}}^2$ (Welsch, 1982; Savšek in Ambrožič, 2023):

$$T_{2D_{c,c+1}}^2 = \frac{dl_{dD_{c,c+1}} Q_{dD_{c,c+1}}^{-1} dl_{dD_{c,c+1}}}{n_D \cdot s^2}, c = 1, \ldots, (Sim - 1), \tag{6}$$

kjer je

$$dl_{dD_{c,c+1}} = D_{ij_{c+1}} - D_{ij_c} \tag{7}$$

sprememba dolžine med točkama P_i in P_j med dvema zaporednima simulacijama,

$$D_{ij_c} = \sqrt{\left(\hat{y}_{j_c} - \hat{y}_{i_c}\right)^2 + \left(\hat{x}_{j_c} - \hat{x}_{i_c}\right)^2} \text{ in } D_{ij_{c+1}} = \sqrt{\left(\hat{y}_{j_{c+1}} - \hat{y}_{i_{c+1}}\right)^2 + \left(\hat{x}_{j_{c+1}} - \hat{x}_{i_{c+1}}\right)^2} \tag{8}$$

sta dolžini med točkama P_i in P_j v simulacijah c in $(c + 1)$, $\hat{y}_{i_c}$, $\hat{x}_{i_c}$ in $\hat{y}_{j_c}$, $\hat{x}_{j_c}$ so izravnane koordinate točk P_i in P_j, zapisane v vektorju $\hat{\mathbf{x}}_c$ v simulaciji c, $\hat{y}_{i_{c+1}}$, $\hat{x}_{i_{c+1}}$ in $\hat{y}_{j_{c+1}}$, $\hat{x}_{j_{c+1}}$ so izravnane koordinate točk P_i in P_j, zapisane v vektorju $\hat{\mathbf{x}}_{c+1}$ v simulaciji $(c + 1)$. Element matrike kofaktorjev spremembe dolžine $Q_{dD_{c,c+1}}$ v enačbi (6) izračunamo z

$$Q_{dD_{c,c+1}} = \mathbf{L}_{dD_{ij}} \mathbf{Q}_{\mathbf{u}dD_{ij}} \mathbf{L}_{dD_{ij}}^{\mathrm{T}}, \tag{9}$$

kjer je vektor parcialnih odvodov meritev

$$\mathbf{L}_{dD_{ij}} = [-\sin \nu_{ij}, \quad -\cos \nu_{ij}, \quad \sin \nu_{ij}, \quad \cos \nu_{ij}] \tag{10}$$

$$\nu_{ij} = (\nu_{ij_c} + \nu_{ij_{c+1}})/2 \tag{11}$$

je srednja vrednost smernih kotov med točkama P_i in P_j med dvema zaporednima simulacijama,

$$\nu_{ij_c} = \arctan \frac{\hat{y}_{j_c} - \hat{y}_{i_c}}{\hat{x}_{j_c} - \hat{x}_{i_c}} \text{ in } \nu_{ij_{c+1}} = \arctan \frac{\hat{y}_{j_{c+1}} - \hat{y}_{i_{c+1}}}{\hat{x}_{j_{c+1}} - \hat{x}_{i_{c+1}}} \tag{12}$$

sta smerna kota med točkama P_i in P_j v simulacijah c in $(c + 1)$. V podmatriki $\mathbf{Q}_{\mathbf{u}dD_{ij}}$ so pripadajoči elementi matrike $\mathbf{Q}_{\mathbf{u}}$, ki se nanašajo na točki P_i in P_j.

Za potrditev pravilnega števila prostostnih stopenj $n_D = 1$ v izračunu testne statistike (6) in izračunu mej območja zavrnitve ničelne domneve moramo izračunane testne statistike $T^2_{2D_{c,c+1}}$ po enačbi (6) razvrstiti po velikosti od najmanjše do največje vrednosti in narisati graf empirične porazdelitvene funkcije.

3.3 Testiranje sprememb kotov v geodetski mreži

Pri testiranju sprememb kotov med točkami v geodetski mreži, kar je v članku Savšek in Ambrožič (2023) predstavljeno kot novost v testiranju preoblikovanja geodetske mreže po postopku München, sestavimo testno statistiko med dvema zaporednima simulacijama $T^2_{2\alpha_{c,c+1}}$ (Savšek in Ambrožič, 2023):

$$T^2_{2\alpha_{c,c+1}} = \frac{dl_{d\alpha_{c,c+1}} Q^{-1}_{d\alpha_{c,c+1}} dl_{d\alpha_{c,c+1}}}{n_\alpha \cdot s^2}, c = 1, \ldots, (Sim - 1), \tag{13}$$

kjer je

$$dl_{d\alpha_{c,c+1}} = \alpha_{ijk_{c+1}} - \alpha_{ijk_c} \tag{14}$$

sprememba kota med točkami P_i, P_j in P_k med dvema zaporednima simulacijama,

$$\alpha_{ijk_c} = \nu_{ik_c} - \nu_{ij_c} \text{ in } \alpha_{ijk_{c+1}} = \nu_{ik_{c+1}} - \nu_{ij_{c+1}} \tag{15}$$

sta kota z vrhom v točki P_i med točkama P_j in P_k v simulacijah c in $(c + 1)$, ν_{ij_c}, ν_{ik_c}, $\nu_{ij_{c+1}}$ in $\nu_{ik_{c+1}}$ so smerni koti, izračunani iz izravnanih koordinat med točkami P_i in P_j ter P_i in P_k v simulacijah c in $(c + 1)$. Element matrike kofaktorjev spremembe kota $Q_{d\alpha_{c,c+1}}$ v enačbi (13) izračunamo z

$$Q_{d\alpha_{c,c+1}} = \mathbf{L}_{d\alpha_{ijk}} \mathbf{Q}_{\mathbf{u}d\alpha_{ijk}} \mathbf{L}_{d\alpha_{ijk}}^{\mathrm{T}}, \tag{16}$$

kjer je vektor parcialnih odvodov meritev

$$\mathbf{L}_{d\alpha_{ijk}}=\left[\left(-\frac{\cos\nu_{ik}}{D_{ik}}+\frac{\cos\nu_{ij}}{D_{ij}}\right),\left(\frac{\sin\nu_{ik}}{D_{ik}}-\frac{\sin\nu_{ij}}{D_{ij}}\right),\left(-\frac{\cos\nu_{ij}}{D_{ij}}\right),\left(\frac{\sin\nu_{ij}}{D_{ij}}\right),\left(\frac{\cos\nu_{ik}}{D_{ik}}\right),\left(-\frac{\sin\nu_{ik}}{D_{ik}}\right)\right], \quad (17)$$

$$\nu_{ij}=(\nu_{ij_c}+\nu_{ij_{c+1}})/2 \text{ in } \nu_{ik}=(\nu_{ik_c}+\nu_{ik_{c+1}})/2 \quad (18)$$

sta srednji vrednosti smernih kotov,

$$D_{ij}=(D_{ij_c}+D_{ij_{c+1}})/2 \text{ in } D_{ik}=(D_{ik_c}+D_{ik_{c+1}})/2 \quad (19)$$

sta srednji vrednosti dolžin iz izravnanih koordinat med točkami P_i in P_j ter P_i and P_k v simulacijah c in $(c+1)$. V podmatriki $\mathbf{Q}_{\mathbf{u}d\alpha_{ijk}}$ so pripadajoči elementi matrike $\mathbf{Q}_{\mathbf{u}}$, ki se nanašajo na točke P_i, P_j in P_k.

Za potrditev pravilnega števila prostostnih stopenj $n_\alpha = 1$ v izračunu testne statistike (13) in izračunu mej območja zavrnitve ničelne domneve moramo izračunane testne statistike $T^2_{2\alpha_{c,c+1}}$ po enačbi (13) razvrstiti po velikosti od najmanjše do največje vrednosti in narisati graf empirične porazdelitvene funkcije.

3.4 Testiranje sprememb koordinat oglišč trikotnikov v geodetski mreži

Pri testiranju sprememb koordinat oglišč trikotnikov v geodetski mreži, ki je eden izmed sestavnih delov testiranja preoblikovanja geodetske mreže po postopku München, sestavimo testno statistiko med dvema zaporednima simulacijama $T^2_{2\Delta_{c,c+1}}$ (Welsch, 1983; Welsch in Zhang, 1983) po enačbi iz Savšek in Ambrožič (2023):

$$T^2_{2\Delta_{c,c+1}}=\frac{\mathbf{u}^{\mathrm{T}}_{\Delta_{c,c+1}}\mathbf{Q}^{-1}_{\mathbf{u}\Delta}\mathbf{u}_{\Delta_{c,c+1}}}{n_\Delta\cdot s^2}, c=1,\ldots,(Sim-1), \quad (20)$$

kjer je

$$\mathbf{u}_{\Delta_{c,c+1}}=\hat{\mathbf{x}}_{c+1}-\hat{\mathbf{x}}_{c} \quad (21)$$

vektor premikov oziroma sprememba koordinat oglišč v trikotniku P_i, P_j in P_k med dvema zaporednima simulacijama. V podmatriki $\mathbf{Q}_{\mathbf{u}\Delta}$ so pripadajoči elementi matrike $\mathbf{Q}_{\mathbf{u}}$, ki se nanašajo na točke P_i, P_j in P_k.

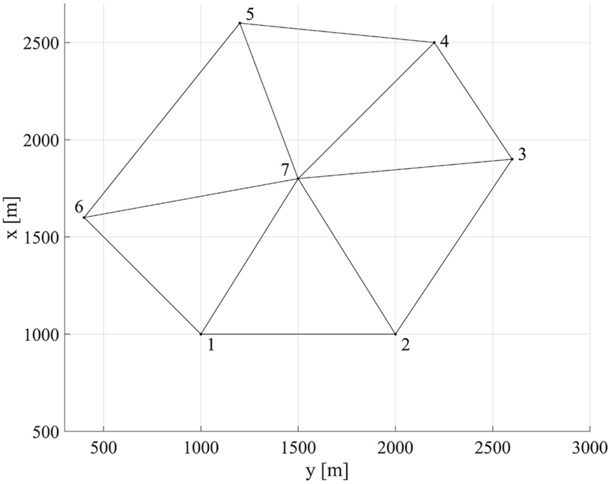
Za potrditev pravilnega števila prostostnih stopenj $n_\Delta = 3$ v izračunu testne statistike (20) in izračunu mej območja zavrnitve ničelne domneve moramo izračunane testne statistike $T^2_{2\Delta_{c,c+1}}$ po enačbi (20) razvrstiti po velikosti od najmanjše do največje vrednosti in narisati graf empirične porazdelitvene funkcije.

```
Začetek
  • Čitanje vhodnih podatkov:
      • izravnane koordinate točk $\hat{\mathbf{x}}_c, \hat{\mathbf{x}}_{c+1}$
      • matrika kofaktorjev koordinatnih neznank $\mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c}, \mathbf{Q}_{\hat{\mathbf{x}}_{c+1}\hat{\mathbf{x}}_{c+1}}$
      • referenčna varianca a posteriori $s_c^2, s_{c+1}^2$
      • število nadštevilnih meritev $f_c, f_{c+1}$
  • Za vse simulacije $c = 1, \ldots, (Sim\text{-}1)$:
      • testiranje skladnosti geodetske mreže:
          • $\mathbf{u}_{c,c+1} = \hat{\mathbf{x}}_{c+1} - \hat{\mathbf{x}}_c$
          • $\mathbf{Q}_{\mathbf{u}} = \mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c} + \mathbf{Q}_{\hat{\mathbf{x}}_{c+1}\hat{\mathbf{x}}_{c+1}} = 2\mathbf{Q}_{\hat{\mathbf{x}}_c\hat{\mathbf{x}}_c}$
          • $f = f_c + f_{c+1}, f_{\mathbf{u}} = \text{rank}\ \mathbf{Q}_{\mathbf{u}} = u - d$
          • $s^2 = \frac{f_c s_c^2 + f_{c+1} s_{c+1}^2}{f_c + f_{c+1}} \ldots (4)$
          • $T^2_{1_{c,c+1}} = \frac{s_{\mathbf{u}}^2}{s^2} = \frac{\mathbf{u}^{\mathrm{T}}_{c,c+1}\mathbf{Q}^{-1}_{\mathbf{u}}\mathbf{u}_{c,c+1}}{f_{\mathbf{u}} \cdot s^2} \ldots (5)$
          • $F_{f_{\mathbf{u}},f} = \text{fcdf}(f_{\mathbf{u}}, f)$
      • Za vse dolžine testiranje sprememb dolžin v geodetski mreži:
          • $dl_{dD_{c,c+1}} = D_{ij_{c+1}} - D_{ij_c} \ldots (7)$
          • $\nu_{ij} = (\nu_{ij_c} + \nu_{ij_{c+1}})/2 \ldots (11)$
          • $\mathbf{L}_{dD_{ij}} = [-\sin\nu_{ij}, \ -\cos\nu_{ij}, \ \sin\nu_{ij}, \ \cos\nu_{ij}] \ldots (10)$
          • $Q_{dD_{c,c+1}} = \mathbf{L}_{dD_{ij}}\mathbf{Q}_{\mathbf{u}dD_{ij}}\mathbf{L}^{\mathrm{T}}_{dD_{ij}} \ldots (9)$
          • $n_D = 3$
          • $T^2_{2D_{c,c+1}} = \frac{dl_{dD_{c,c+1}} Q^{-1}_{dD_{c,c+1}} dl_{dD_{c,c+1}}}{n_D \cdot s^2} \ldots (6)$
          • $F_{n_D,f} = \text{fcdf}(n_D, f) = \text{fcdf}(1, f)$
      • Konec dolžin v geodetski mreži
      • Za vse kote testiranje sprememb kotov v geodetski mreži:
          • $\alpha_{ijk_c} = \nu_{ik_c} - \nu_{ij_c}, \alpha_{ijk_{c+1}} = \nu_{ik_{c+1}} - \nu_{ij_{c+1}} \ldots (15)$
          • $dl_{d\alpha_{c,c+1}} = \alpha_{ijk_{c+1}} - \alpha_{ijk_c} \ldots (14)$
          • $\nu_{ij} = (\nu_{ij_c} + \nu_{ij_{c+1}})/2, \ \nu_{ik} = (\nu_{ik_c} + \nu_{ik_{c+1}})/2 \ldots (18)$
          • $D_{ij} = (D_{ij_c} + D_{ij_{c+1}})/2, D_{ik} = (D_{ik_c} + D_{ik_{c+1}})/2 \ldots (19)$
          • $\mathbf{L}_{d\alpha_{ijk}} = \left[\left(-\frac{\cos\nu_{ik}}{D_{ik}} + \frac{\cos\nu_{ij}}{D_{ij}}\right), \left(\frac{\sin\nu_{ik}}{D_{ik}} - \frac{\sin\nu_{ij}}{D_{ij}}\right), \left(-\frac{\cos\nu_{ij}}{D_{ij}}\right), \left(\frac{\sin\nu_{ij}}{D_{ij}}\right), \left(\frac{\cos\nu_{ik}}{D_{ik}}\right), \left(-\frac{\sin\nu_{ik}}{D_{ik}}\right)\right] \ldots (17)$
          • $Q_{d\alpha_{c,c+1}} = \mathbf{L}_{d\alpha_{ijk}}\mathbf{Q}_{\mathbf{u}d\alpha_{ijk}}\mathbf{L}^{\mathrm{T}}_{d\alpha_{ijk}} \ldots (16)$
          • $n_\alpha = 1$
          • ${}_{c,c\ \ 1} = \frac{dl_{d\alpha_{c,c+1}} Q_{d\alpha_{c,c+1}} dl_{d\alpha_{c,c+1}}}{n \ \ \cdot s} \quad (13)$
          • $F_{n_\alpha,f} = \text{fcdf}(n_\alpha, f) = \text{fcdf}(1, f)$
      • Konec kotov v geodetski mreži
      • Za vse trikotnike testiranje sprememb koordinat oglišč trikotnikov v geodetski mreži:
          • $\mathbf{u}_{\Delta_{c,c+1}} = \hat{\mathbf{x}}_{c+1} - \hat{\mathbf{x}}_c \ldots (21)$
          • $n_\Delta = 1$
          • $T^2_{2\Delta_{c,c+1}} = \frac{\mathbf{u}^{\mathrm{T}}_{\Delta_{c,c+1}}\mathbf{Q}^{-1}_{\mathbf{u}\Delta}\mathbf{u}_{\Delta_{c,c+1}}}{n_\Delta \cdot s^2} \ldots (20)$
          • $F_{n_\Delta,f} = \text{fcdf}(n_\Delta, f) = \text{fcdf}(3, f)$
      • Konec trikotnikov v geodetski mreži
      • Izris porazdelitvenih funkcij
  • Konec simulacij
Konec
```

Slika 2: Diagram poteka simulacij za določitev števila prostostnih stopenj.

4 Računski primer

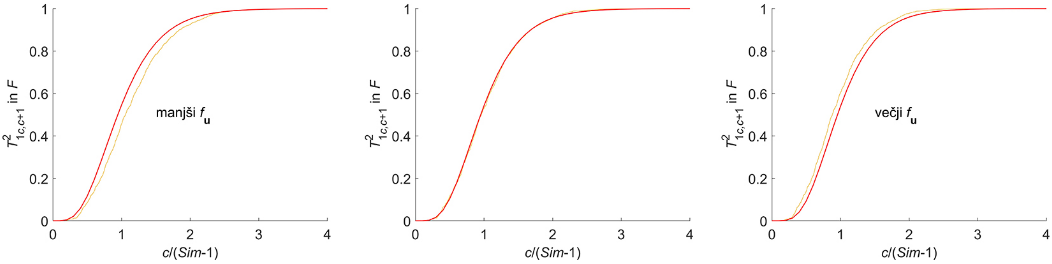
Za potrditev pravilnega števila prostostnih stopenj pri izračunu mej območja zavrnitve ničelne domneve obravnavamo primer 2D geodetske mreže, ki smo ga uporabili za prikaz delovanja vseh postopkov deformacijske analize (slika 3). Primer obravnavane 2D geodetske mreže smo obravnavali v naši raziskovalni skupini v več znanstvenih člankih (Ambrožič, 2001, 2004; Ambrožič et al., 2019; Hamza et al., 2020; Marjetič et al., 2012; Savšek in Ambrožič, 2023; Soldo in Ambrožič, 2018; Vrečko in Ambrožič, 2013) ter v Batilović et al. (2022). Pravilno izbrano število prostostnih stopenj bomo pokazali na primerih izračuna kritičnih vrednosti pri testiranju skladnosti, pri testiranju sprememb dolžin in testiranju sprememb koordinat oglišč trikotnikov v geodetski mreži.



Slika 3: Skica mreže.

4.1 Število prostostnih stopenj pri testiranju skladnosti

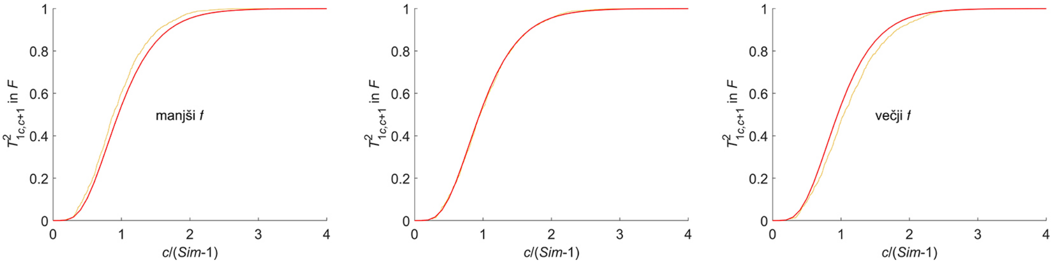
Pri testiranju skladnosti geodetske mreže izračunamo testne statistike $T^{2}_{1_{c,c+1}}$, $c = 1, \ldots, (Sim - 1)$ po enačbi (5) in jih razvrstimo po velikosti od najmanjše do največje. Narišemo graf tako, da na absciso nanesemo vrednosti $T^{2}_{1_{c,c+1}}$, na ordinato pa $c/(Sim - 1)$, $c = 1, \ldots, (Sim - 1)$. Na sliki 4 prikazujemo grafe porazdelitvene funkcije testne statistike za različne vrednosti prvega parametra prostostnih stopenj $f_{\mathbf{u}}$ ter porazdelitveno funkcijo porazdelitve F pri pravilnem številu prostostnih stopenj $f_{\mathbf{u}}$ in f.



Slika 4: Grafi porazdelitvene funkcije testne statistike pri testiranju skladnosti geodetske mreže (gladka tanka rjava črta) za *Sim* = 1000 simulacij in porazdelitvena funkcija porazdelitve F (gladka odebeljena rdeča črta).

Slika 4 jasno prikazuje, da graf porazdelitvene funkcije testne statistike (5) popolnoma sovpada z grafom porazdelitvene funkcije porazdelitve F za prvi parameter prostostnih stopenj $f_{\mathbf{u}} = u - d$ (slika 4 sredina), kar pa ni primer, ko prvo število $f_{\mathbf{u}}$ zmanjšamo ($f_{\mathbf{u}} = u - d - 1$; slika 4 levo) ali zvečamo za ena ($f_{\mathbf{u}} = u - d + 1$; slika 4 desno).

Slika 5 prikazuje grafe porazdelitvene funkcije testne statistike za različne vrednosti drugega parametra prostostnih stopenj ter porazdelitveno funkcijo porazdelitve F pri pravilnem številu prostostnih stopenj $f_{\mathbf{u}}$ in f.

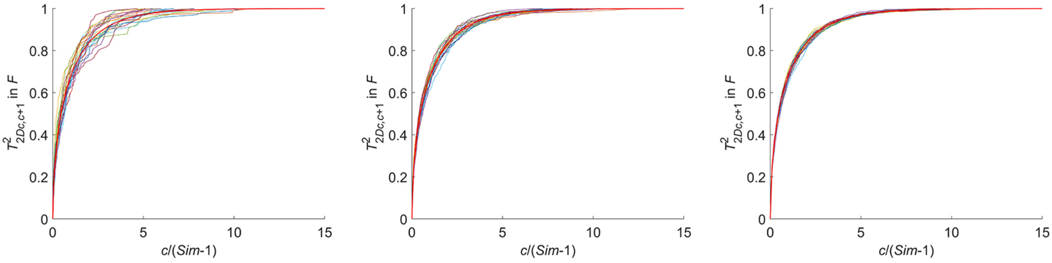


Slika 5: Grafi porazdelitvene funkcije testne statistike pri testiranju skladnosti geodetske mreže (gladka tanka rjava črta) za *Sim* = 1000 simulacij in porazdelitvena funkcija porazdelitve F (gladka odebeljena rdeča črta).

Slika 5 jasno prikazuje, da grafi porazdelitvene funkcije testne statistike (5) popolnoma sovpadajo z grafom porazdelitvene funkcije porazdelitve F za drugi parameter prostostnih stopenj $f = f_1 + f_2$ (slika 5 sredina), kar pa se ne zgodi, ko drugo število zmanjšamo ($f = f_1 + f_2 - 5$; slika 5 levo) ali povečamo za pet ($f = f_1 + f_2 + 5$; slika 5 desno). Če bi število prostostnih stopenj zmanjšali ali povečali za ena, bi bila razlika manj opazna.

4.2 Število prostostnih stopenj pri testiranju sprememb dolžin

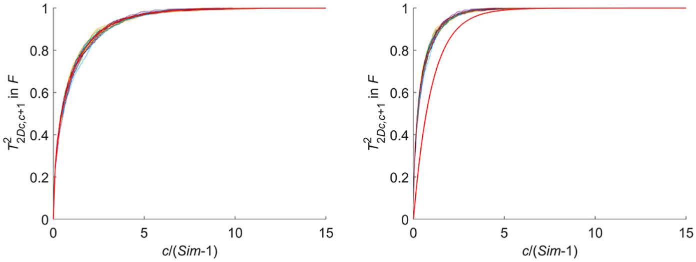
Pri testiranju sprememb dolžin v geodetski mreži izračunamo testne statistike $T^2_{2D_{c,c+1}}$, $c = 1, \ldots, (Sim - 1)$ po enačbi (6) in jih razvrstimo po velikosti od najmanjše do največje. Narišemo graf tako, da na absciso nanesemo vrednosti $T^2_{2D_{c,c+1}}$, na ordinato pa $c/(Sim - 1)$, $c = 1, \ldots, (Sim - 1)$. Na sliki 6 prikazujemo grafe empirične porazdelitvene funkcije testne statistike z različnim številom simulacij testnih statistik za vseh 21 dolžin med točkami pri enakem številu prostostnih stopenj ($n_D = 1$).



Slika 6: Grafi porazdelitvene funkcije testne statistike (6) za različno število simulacij.

Slika 6 prikazuje, da grafi porazdelitvene funkcije testne statistike že pri 100 simulacijah dobijo pravilno obliko (slika 6 levo), pri 500 simulacijah so grafi že zelo podobni ne glede na to, katero dolžino obravnavamo (slika 6 sredina), pri 1000 simulacijah pa grafi za vse razdalje sovpadajo (slika 6 desno).

Za potrditev pravilnega števila prostostnih stopenj prikažemo na sliki 7 grafe empirične porazdelitvene funkcije testne statistike (6) za vseh 21 dolžin in graf porazdelitvene funkcije porazdelitve F pri različnih številih prostostnih stopenj.



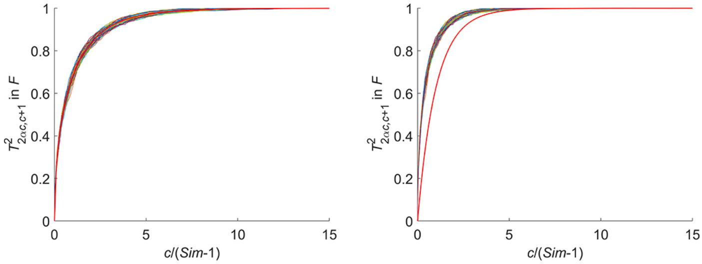
Slika 7: Grafi porazdelitvene funkcije testne statistike za spremembe dolžin (črte različnih barv) za *Sim* = 1000 simulacij in z različnim številom prostostnih stopenj ter porazdelitvena funkcija porazdelitve F (gladka odebeljena rdeča črta).

Slika 7 jasno prikazuje, da se grafi porazdelitvene funkcije testne statistike (6) natančno ujemajo z grafom porazdelitvene funkcije porazdelitve F pri številu prostostnih stopenj $n_D = 1$ (slika 7 levo), kar pa se ne zgodi pri številu prostostnih stopenj $n_D = 2$ (slika 7 desno).

4.3 Število prostostnih stopenj pri testiranju sprememb kotov

Pri testiranju sprememb kotov v geodetski mreži izračunamo testne statistike $T^2_{2\alpha_{c,c+1}}$, $c = 1, \ldots, (Sim - 1)$ po enačbi (13) in jih razvrstimo po velikosti od najmanjše do največje. Narišemo graf tako, da na absciso nanesemo vrednosti $T^2_{2\alpha_{c,c+1}}$, na ordinato pa $c/(Sim - 1)$, $c = 1, \ldots, (Sim - 1)$.

Za potrditev pravilnega števila prostostnih stopenj prikažemo na sliki 8 grafe empirične porazdelitvene funkcije testne statistike (13) za vseh 105 kotov in graf porazdelitvene funkcije porazdelitve F pri različnih številih prostostnih stopenj.



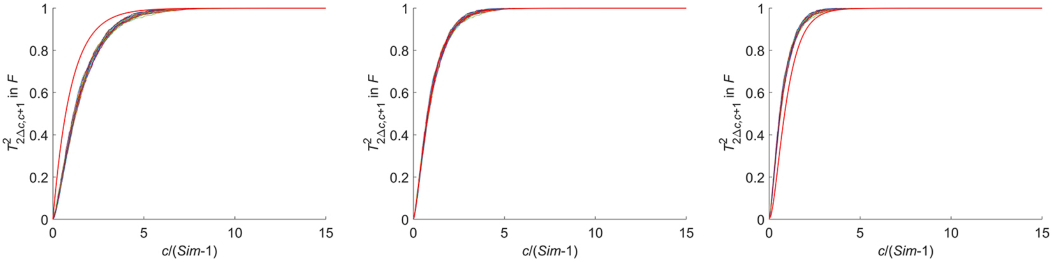
Slika 8: Grafi empirične porazdelitvene funkcije testne statistike za spremembe kotov (črte različnih barv) za *Sim* = 1000 simulacij in z različnim številom prostostnih stopenj ter porazdelitvena funkcija porazdelitve F (gladka odebeljena rdeča črta).

Slika 8 jasno prikazuje, da se grafi empirične porazdelitvene funkcije testne statistike (13) natančno ujemajo z grafom porazdelitvene funkcije porazdelitve F pri številu prostostnih stopenj $n_\alpha = 1$ (slika 8 levo), kar pa se ne zgodi pri številu prostostnih stopenj $n_\alpha = 2$ (slika 8 desno).

4.4 Število prostostnih stopenj pri testiranju sprememb koordinat oglišč trikotnikov

Pri testiranju sprememb koordinat oglišč trikotnikov v geodetski mreži izračunamo testne statistike $T^2_{2\Delta_{c,c+1}}$, $c = 1, \ldots, (Sim - 1)$ po enačbi (20) in jih razvrstimo po velikosti od najmanjše do največje. Narišemo graf tako, da na absciso nanesemo vrednosti $T^2_{2\Delta_{c,c+1}}$, na ordinato pa $c/(Sim - 1)$, $c = 1, \ldots, (Sim - 1)$.

Za potrditev pravilnega števila prostostnih stopenj prikažemo na sliki 9 grafe empirične porazdelitvene funkcije testne statistike (20) za vseh 35 trikotnikov in graf porazdelitvene funkcije porazdelitve F pri različnih številih prostostnih stopenj.



Slika 9: Grafi empirične porazdelitvene funkcije testne statistike za spremembe koordinat oglišč trikotnikov (črte različnih barv) za *Sim* = 1000 simulacij in z različnim številom prostostnih stopenj ter porazdelitvena funkcija porazdelitve F (gladka odebeljena rdeča črta).

Slika 9 jasno prikazuje, da se grafi empirične porazdelitvene funkcije testne statistike (20) natančno ujemajo z grafom porazdelitvene funkcije porazdelitve F pri številu prostostnih stopenj $n_\Delta = 3$ (slika 9 sredina), kar pa se ne zgodi pri številu prostostnih stopenj $n_\Delta = 2$ (slika 9 levo) ali $n_\Delta = 4$ (slika 9 desno).

S poskusi na slikah 4–9 potrjujemo, da lahko pravilno število prostostnih stopenj določimo tudi z metodo Monte Carlo.

5 Razprava in zaključki

V naši raziskavi smo prikazali alternativno uporabo metode Monte Carlo, in sicer za potrditev pravilnega števila prostostnih stopenj za izračun meje območja zavrnitve ničelne domneve. Najprej smo simulirali meritve horizontalnih smeri in horizontalnih dolžin v 2D geodetski mreži ter jih izravnali po metodi najmanjših kvadratov. Simulacije meritev in izravnave mreže smo ponovili 1000-krat. Vrednost 1000 simulacij smo izbrali zato, ker so pri tej vrednosti grafi empirične porazdelitvene funkcije testne statistike skoraj popolnoma sovpadali z grafom porazdelitvene funkcije porazdelitve F. Med dvema zaporednima izravnavama mreže, ki predstavljata dve terminski izmeri, smo izračunali testne statistike in jih uporabili pri testiranju skladnosti mreže, pri testiranju sprememb dolžin in testiranju sprememb kotov ter pri testiranju sprememb koordinat oglišč trikotnikov v geodetski mreži. Testne statistike za vsako testirano količino smo razvrstili po velikosti od najmanjše do največje vrednosti in jih prikazali na grafu skupaj s porazdelitveno funkcijo porazdelitve F, po kateri so testne statistike porazdeljene.

Cilj naše raziskave je bil empirično potrditi verjetnostno porazdelitev in število prostostnih stopenj pri testiranju skladnosti geodetske mreže, sprememb dolžin, kotov in koordinat oglišč trikotnikov, kot je predlagano v deformacijski analizi po postopku München. Naši rezultati potrjujejo, da:

1. je testna statistika za testiranje skladnosti geodetske mreže med dvema izmerama porazdeljena po porazdelitvi F ($F_{f_\mathbf{u},f}$), torej s prostostnimi stopnjami $f_\mathbf{u}$ in f,
2. je testna statistika za testiranje spremembe dolžine med dvema točkama P_i in P_j med dvema izmerama porazdeljena po porazdelitvi F ($F_{1,f}$), torej s prvim številom prostostnih stopenj $n_D = 1$,
3. je testna statistika za testiranje spremembe koordinat oglišč v trikotniku P_i, P_j in P_k med dvema izmerama porazdeljena po porazdelitvi F ($F_{3,f}$), torej s prvim številom prostostnih stopenj $n_\Delta = 3$.

Poleg tega smo ugotovili, da:

4. je testna statistika za testiranje spremembe kotov med točkami P_i, P_j in P_k med dvema izmerama porazdeljena po porazdelitvi F ($F_{1,f}$), torej s prvim številom prostostnih stopenj $n_\alpha = 1$.

V literaturi nismo zasledili teh vrednosti, zato menimo, da je to najpomembnejši prispevek naše raziskave. Pravilno določeno število prostostnih stopenj je ključno za izračun meje območja zavrnitve ničelne domneve pri testiranju različnih testnih statistik v postopku München in je zato odločilnega pomena za sprejemanje pravilnih odločitev pri analizi izmerjenih deformacij različnih objektov.

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